MA 16020 Lesson 29: Double integrals III

Recall (geom. interpretation of double integrals):
Given a function $z = f(x, y)$ of two variables and a region R in the xy–plane, the integral $\iint_R f(x, y)\mathrm{d}A$ has the meaning of:

Exercise 1. Compute the volume of the solid bounded by the surface $z = e^x\sqrt{y^3 + e^x}$ from above, by the xy–plane from below and by the planes $x = -1, x = 2, y = 0$ and $y = 2$ on the sides.

Exercise 2. Compute the volume under the surface $z = x^2y$ and above the triangle with vertices $(1,1,0), (1,5,0)$ and $(4,1,0)$.

Exercise 3. There is a heater in a corner of a rectangular room of dimensions 8×10 m. As a result, the temperature in $^\circ C$ in the room is described by

$$T(x, y) = 60 - 0.3(x^2 + y^2),$$

where (x, y) are the coordinates of a given point in the room (the heater is placed at $(0, 0)$). What is the average temperature in the room?

Exercise 4. The water temperature in a lake during the night is given approximately (in $^\circ F$) by the function

$$T(d,t) = \frac{350e^{-0.05t}}{d+5}$$

where t is the number of hours that passed since 8 pm and d is the depth in m. What is the average temperature of the water from the surface to the depth of 5 m and between 10 pm and 1 am?