MA 16020 Lesson 27: Double integrals I

Let $z = f(x, y)$ be a function of two variables. Similarly to taking (partial) derivatives with respect to x and y, we can take

1. $\int f(x, y)\mathrm{d}x \ldots$

2. $\int f(x, y)\mathrm{d}y \ldots$

We will be mostly concerned with definite integrals. Combining the above two ways of integration, we obtain a double integral:

$$\int_a^b \int_c^d f(x, y)\mathrm{d}x\mathrm{d}y =$$

$$\int_a^b \int_c^d f(x, y)\mathrm{d}y\mathrm{d}x =$$

Recall that for a function $g(x)$ of one variable, the meaning of the integral $\int_a^b g(x)\mathrm{d}x$ is:

Similarly, the geometric meaning of the double integral $\int_a^b \int_c^d f(x, y)\mathrm{d}x\mathrm{d}y$ is:

Exercise 1. Evaluate the integral

$$\int_2^4 \int_0^5 \frac{x^3}{y^2} \mathrm{d}x \mathrm{d}y \ .$$

Exercise 2. Evaluate the integral

$$\int_{-1}^2 \int_0^{\pi/3} 2y^3 \sin(x) \mathrm{d}x \mathrm{d}y \ .$$

Sometimes the limits of the "inner integral" may depend on the unused variable. This corresponds to integration over regions that are not necessarily rectangular.

Example.

$$\int_1^3 \int_1^x x^2 y \mathrm{d}y \mathrm{d}x$$

Exercise 3. Evaluate the integral

$$\int_0^{\sqrt{\pi/6}} \int_{-y^2}^{0} y\cos(x)\mathrm{d}x\mathrm{d}y \ .$$