MA 16020 Lesson 26: Lagrange multipliers II

Recall (constrained min/max using Lagrange multipliers):
When trying to minimize/maximize the value of the function $z = f(x, y)$ subject to the constraint $g(x, y) = C$, the critical points are given as points (x, y) that are solutions to the system of the equations:

as well as the original constraint:

typical strategy to solve the system (not always):

Exercise 1. The prices for constructing a box with a sqaure base are: \$15 per m^3 for the top, \$12 per m^3 for the bottom, and \$6 per m^3 for the sides. What is the biggest possible volume of the box for \$100?

Exercise 2. Given that a company spends x thousands of dollars on internet advertising and y thousands of dollars on other forms of advertising, it is expected to sell

$$S(x.y) = 15000 + 100x^{1.5}y^{0.5}$$

units of their product. How should the company distribute its advertising budget of \$200 000 to achieve maximum sales?

Exercise 3. In a certain region, the population of rabbits R (in hundreds of specimens) and the population of foxes F (in hundreds of specimens) satisfy the equation

$$2(R-15)^2+6(F-10)^2=144.$$

What is the maximal and minimal possible combined number of rabbits and foxes living in the region?

Exercise 4. If a certain strain of bacteria is fed by x grams of nutrient A, y grams of nutrient B, it will ultimately produce $x^{0.6}y^{0.4}$ grams of a desired chemical. The cost of the nutrients are: 15 dollars per gram for nutrient A and 11 dollars per gram of nutrient B. What is the minimal cost to produce 50 grams of the desired chemical?