MA 16020 Lesson 24: Extrema of functions of two variables II

Recall (extrema of a function of two variables):
To find local extrema of a function $z = f(x, y)$ of two varables, we need to

1. Find all the *critical points:* Points (x, y) satisfying:

2. Compute all the second-order partial derivatives of f and $D =$

3. For a given critical point (x_0, y_0), evaluate D and f_{xx} at (x_0, y_0).
   If ________________________, then (x_0, y_0) is a local minimum of f.
   If ________________________, then (x_0, y_0) is a local maximum of f.
   If ________________________, then (x_0, y_0) is a saddle point of f.
   If ____________________, then the test is inconclusive for this (x_0, y_0).

Exercise 1. Find all the critical points of the function

$$f(x, y) = x^2y - 2x^2 - 3y^2 + 3y - 7 \ .$$

Exercise 2. A shop provides two brands of shoes. The acquiring cost is 5 dollars per pair for the first brand and 4 dollars per pair for the second brand. If the selling prices are x dollars per pair of shoes of the first brand and y dollars per pair of shoes of the second brand, it is expected that the customers will buy approximately and $75 + y - 2x$ pairs of shoes of the first brand and $50 + x - 2y$ pairs of shoes of the second brand. Find the optimal selling prices and maximal profit.

Exercise 3. A rectangular box of volume 3 m^3 is to be made. The cost of material is: 25 dollars per m^3 for the bottom, 15 dollars per m^3 for the sides, and 20 dollars per m^3 for the top. Find the dimensions of the box so that the cost is minimal, and the cost of the box.

Exercise 3. If a certain strain of bacteria is fed by x grams of nutrient A, y grams of nutrient B and z grams of nutrient C, it will ultimately produce x^3y^3z grams of a desired chemical. The cost of the nutrients are: 15 dollars per gram for nutrient A, 5 dollars per gram of nutrient B and 2 dollars per gram of nutrient C. What is the minimal cost to produce 500 grams of the desired chemical?