MA 16020 Lesson 22: Chain rule for multivariate functions

When $z = f(x, y)$ is a function of x and y, and $x = g(t), y = h(t)$ are further functions of a common variable t, the overall function $z = f(g(t), h(t))$ is a function of one variable. While its derivative $\mathrm{d}z/\mathrm{d}t$ can be computed directly, it is often benefitial to use the *chain rule for functions of two variables:*

Example: If $z = x/y$, $x = \ln(t), y = t^3 + t^2 + 1$, we may compute $\mathrm{d}z/\mathrm{d}t$

(a) using the chain rule:

(b) directly:

Let us verify that the result is the same:

Exercise 1. Compute $\frac{\mathrm{d}z}{\mathrm{d}t}$ when $z = 3x^2y^3$, $x = \sin(3t+1)$ and $y = e^{2t} - 2$

Exercise 2. Evaluate $\frac{\mathrm{d}z}{\mathrm{d}t}$ at $t = 2$ when $z = \sin(xy)$, $x = \frac{\pi t^2}{4}$ and $y = \frac{t}{4}$.

Exercise 3. The number of units of a certain product sold per month is given by the function

$$S(x, y) = 3y^2 + 2yx + x,$$

where x is the amount spent on advertising per month (in thousands of dollars) and y is the amount spent on distribution per month (in thousands of dollars). Given that t months from now, the monthly amount spent on advertising is $15 + t$ thousands of dollars and the monthly amount spent on distribution is $t^2 + t + 3$ thousands of dollars, find the rate of change of the number of units sold per month 4 months from now.

Exercise 4. The radius of the base of a cylinder is decreasing at the rate $3\,\text{mm/s}$ while its height is increasing at the rate $7\,\text{mm/s}$. What is the rate of change of the volume of the cylinder at the moment when the radius is $40\,\text{mm}$ and the height is $85\,\text{mm}$?