MA 16020 Lesson 21: Differentials of multivariate functions

Recall (differentials of functions of one variable): Given a function $y = f(x)$, small changes in $y = f(x)$ due to small changes of x can be approximated using the derivative:

This becomes an equality of *differentials*:

If $z = f(x, y)$ is a function of two variables, we can consider small changes $\Delta x, \Delta y$ in x *and* in y, and express Δz using the partial derivatives:

Exercise 1. Using differentials, estimate

$$\cos(\pi/4 + 0.02)\sin(\pi/4 - 0.03).$$

Exercise 2. A cylindrical can has base of radius 4 cm and height 12 cm. If the radius is decreased by 0.5 cm, and the height is increased by 1.5 cm, use calculus to estimate the change of the volume of the can.

Exercise 3. A police radar gun measured that a car traveled the distance 52 meters in 2.5 seconds. Assuming that the maximum error in measurement of distance is 0.2 meters and the maximum error in measurement of the time is 0.1 seconds, estimate the maximum error in calculating the speed of the car.

Exercise 4. The pressure (in Pa) of a certain gas in a container is given by the equation

$$P = 160\frac{T}{V},$$

where T is the temperature of the gas (in $^\circ K$) and V is the volume of the container (in m^3). The volume of the container is measured to be $4\mathrm{m}^3$, with the maximum error of measurement $0.2\mathrm{m}^3$, and the temperature of the gas is measured to be $310^\circ K$, with the maximum error of measurement $4^\circ K$. Find the relative percentage error in computing the pressure of the gas.