MA 16020 Lesson 20: Higher partial derivatives

When $z = f(x, y)$ is a function of two variables, so are the partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$. Upon taking partial derivatives of these two functions, we obtain four *second–order partial derivatives* of f:

Fact: While this is not true in full generality, for all functions we encounter, we have

$$f_{xy} = f_{yx}.$$

Exercise 1. Compute all the second–order partial derivatives for

$$f(x, y) = y^3 x e^{xy} .$$

Exercise 2. Compute f_{uu} and f_{uv} for

$$f(u,v) = \sqrt{u^2 + v^4 + 2}\ .$$

Exercise 3. Compute $f_{yy}(1,2)$ when

$$f(x,y) = \ln(2x^3 + 3xy + y)\ .$$

Exercise 4. Compute $f_{xy}(1,3)$ when

$$f(x,y) = 3y^2\ln(x) + \frac{\sqrt{e^{3x} + \ln(x^3+2)}}{5\sqrt[3]{\sin^2(x-4)+1}} + 2yx^3 \ .$$

Exercise 5. Compute all the second–order partial derivatives of

$$f(u,v) = \cos(3u)\sin(4uv) \ .$$