MA 16020 Lesson 19: Partial derivatives

For a function $z = f(x, y)$ of two variables, we have two ways to take derivatives:

The (first) **partial derivative** $\frac{\partial f}{\partial x}$ (or $\frac{\partial z}{\partial x}$, f_x) describes the rate of change of z as ____ changes and ____ remains constant. It is computed as a derivative of f as a function of ____ where we treat the variable ____ as a constant.

The (first) **partial derivative** $\frac{\partial f}{\partial y}$ (or $\frac{\partial z}{\partial y}$, f_y) describes the rate of change of z as ____ changes and ____ remains constant. It is computed as a derivative of f as a function of ____ where we treat the variable ____ as a constant.

Example: Compute the first partial derivatives of the function

$$f(x, y) = x^2 + xy + 5\ln(y) .$$

Recall: The graph of a function of one variable $f(x)$ at a given x_0 has a tangent line, whose slope is dictated by ________________________________.

The graph of a function of two variables $f(x, y)$ at a given (x_0, y_0) has a tangent ______________________.

It can be determined by the partial derivatives:

$\frac{\partial f}{\partial x}(x_0, y_0) =$

$\frac{\partial f}{\partial y}(x_0, y_0) =$

Exercise 1. Compute $f_x.f_y$ when

$$f(x,y) = \frac{3xy}{\sqrt{xy-1}}\ .$$

Exercise 2. Compute $f_x(1,3)$ when

$$f(x,y) = \frac{\ln(3xy+3)}{x+y}\ .$$

Exercise 3. The pressure (in Pa) of certain gas in a container is decribed by the equation

$$P = 50\frac{T}{V}$$

where T is the temperature of the gas (in $^\circ K$) and V is the volume of the container (in m^3). If the temperature of the gas is $320^\circ K$ and the gas is kept in a container of volume $5\,\mathrm{m}^3$, find the rate of change of the pressure both with respect to the change of temperature and with respect to the change of volume.

Exercise 4. A company makes products A and B. If it produces x units of product A and y units of product B, the expected revenue is

$$R(x, y) = 5x + 10y + 3xy \ .$$

If the company makes 15 units of product A and 10 units of product B, find the marginal profits (=rates of change with respect to change of production of product A and product B, resp.)