# MA 16020 Lesson 18: Functions of several variables

A function of *one variable* takes as an input ________________________
and produces as an output ________________________.
A graph of such a function is ________________________________.
**Example:** $f(x) = x^2 + 1$

A function of *two variables* takes as an input ________________________
and produces as an output ________________________.
A graph of such a function is ________________________________.
**Example:** $f(x, y) = x^2 + y^2$

A function of *three variables* takes as an input ________________________
and produces as an output ________________________.
**Example:** $f(x, y, z) = 2x^2 - xyz^2$

... and so on.
(We will stick to functions of two variables for the most part.)

Just as for functions of one variable, we consider for functions of two (or three etc.) variables their

**domain** =

and **range** =

**Example:** Find the domain and the range of the function

$$f(x, y) = \sqrt{2x^2 - 3y} + 5 \ .$$

The graph of a function $f(x, y)$ of two variables is sometimes analyzed via the so-called
**level curves** $=$

**Example:** Find and sketch the level curves of the function

$$f(x, y) = e^{x^2+y^2}$$

for $C = 8$ and $C = e^4$.

**Exercise 1.** Describe the level curves of

$$f(x, y) = 3x^3y.$$

**Exercise 2.** Find the domain of

$$f(x, y) = \frac{\sqrt{3x + y - 1} \cdot \sqrt{3x + y - 2}}{\ln(y - 3) - 4}\ .$$