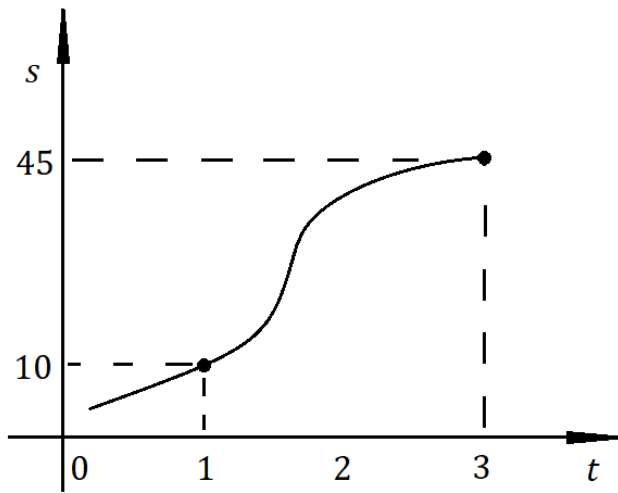


MA 16010 Lesson 8: Instantaneous rate of change

Average vs. instantaneous rate of change. Suppose that $y = f(x)$ is a function, consider some x and some change in x , $\Delta x = h$:

1. The slope of the **secant** line is computed as _____. Its meaning is _____
(=how fast will f grow from x to $x + h$ on average).
2. The slope of the **tangent** line is computed as _____. Its meaning is _____
(=how fast f grows at x , or how fast f grows really close to x).

Example by picture: The following is the graph of distance a car traveled (in m) with respect to time (in s). A radar gun measures the distance at $t = 1$, and then $t = 3$, and estimates the speed. The speed limit is 50 mph ≈ 22.35 m/s. Is the car speeding at any point? Will the radar notice?



Exercise: Compute the rate of change of the area of a square depending on the length of its side l , when $l = 3.5$ m.

Exercise: Consider a pendulum on a train. Its position in the horizontal direction (in m) is described as the function of time (in s) by:

$$s(t) = \frac{1}{2} \sin(t) + 53t$$

- (a) Describe its velocity $v(t)$ (in the hor. direction) as a function of time.
- (b) What is its average speed over the first 10 seconds?

Exercise: We throw a ball vertically in the air, and as a result, its position function (in m, depending on time in s) is:

$$s(t) = 9t - t^2$$

- (a) What is its velocity function?
- (b) At what time does the ball reach its highest point, and how high is it?
- (c) At what time does the ball hit the ground, and with what speed?

Exercise (time permitting): A company's expected profit P (in thousands of dollars) is estimated to be dependent on the amount a of money spent of advertisement (in thousands of dollars) as follows:

$$P(a) = 200\sqrt{a} - a^2 - a$$

(assuming $0 \leq a \leq 130$).

- (a) What is the rate of change of the profit if the company spends $a = 25$ thousands of dollars on advertising?
- (b) What is the rate of change of the profit if the company spends $a = 100$ thousands of dollars on advertising?