MA 16010 Lesson 7: Basic rules of differentiation

Recall: The derivative of $y = f(x)$ at x is defined via limits as:

$\Big($other notation for derivatives: $\Big)$

Today we look at practical rules for computing derivatives.

0. Constant rule: If $f(x) = c$ is a constant function, then $f'(x) =$ ______.

Justification:

1. Power rule: We have $(x^n)' =$

Examples:

- $f(x) = x^0 =$ ______ :

- $f(x) = x^1 = x$:

- $f(x) = x^2$:

Note: The rule works not only for n from non–negative integers, but for **all** exponents, including negative, rational, irrational, ... numbers.

Examples:

2. Trig and exponential functions: We have

$$(\sin(x))' = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_,$$

$$(\cos(x))' = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_,$$

$$(e^x)' = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

3. Sum, difference, constant multiple rules: If $f(x)$, $g(x)$ are functions and c is a constant (i.e. a number), we have:

$$(f(x) + g(x))' = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \quad \text{(Sum rule)},$$

$$(f(x) - g(x))' = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \quad \text{(Difference rule)},$$

$$(c \cdot f(x))' = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \quad \text{(Constant multiple rule)}.$$

Exercise: Find $f'(x)$ when

1. $f(x) = 2x^5 - 3x^2 + 7$:

2. $f(x) = \frac{\sqrt[3]{x^2} - 4x^{-1/5}}{\sqrt{x}} + 2\sin(x)$:

Exercise: Compute $\frac{\mathrm{d}y}{\mathrm{d}x}\Big|_{x=2}$ when $y = \frac{2}{x^3} + 7e^x + 10e^2$.

Exercise: Find the equation of the tangent line to the graph of
$f(x) = 4 + 2\cos(x) \quad \text{at} \quad x = \pi/3$: