MA 16010 Lesson 3: Limits Analytically

Preview: Continuity

A function $f(x)$ is continuous at $x = c$ if:

- ______________________________,
- ______________________________,
- ______________________________.

Rule of thumb: Functions defined by a single formula are continuous at every point where they are defined
(if they are also defined around the point).

Examples:

- $f(x) = \frac{1}{2}x + 2$:

- $f(x) = \sqrt{x}$:

- $f(x) = \frac{1}{x}$:

We can use this implicit continuity to compute simple limits quickly:

Example: Compute:

$$\lim_{x \to -1} \frac{x-1}{x-2}$$

Computational rules for limits.
Assuming that $\lim_{x\to c} f(x)$, $\lim_{x\to c} g(x)$ exist, we have:

- $\lim_{x\to c} (f(x) \pm g(x)) =$
- $\lim_{x\to c} (f(x) \cdot g(x)) =$
- $\lim_{x\to c} (f(x)/g(x)) =$
- $\lim_{x\to c} (k \cdot f(x)) =$
- $\lim_{x\to c} (f(x)^n) =$

as long as the expressions on the right–hand side make sense.

(**Example:**
makes sense:

does not make sense:

)

Example: Using that $\lim_{x\to 0} \frac{\sin(x)}{x} =$ ______, compute:

$$\lim_{x\to 0} \left(\frac{x+2}{x} \cdot \sin(x) + 4x \right)$$

More complicated limits:

1. Type "$0/0$":

Strategy:

Example:

$$\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 - 2x}$$

2. Type "(finite number)/0":

Strategy:

Example:

$$\lim_{x \to 3} \frac{3x + 2}{x - 3}$$

Example:

$$\lim_{x \to 3} \frac{3x+2}{(x-3)^2}$$

3. Function defined piecewise by formulas:

Example: Consider $f(x)$ defined by

$$f(x) = \begin{cases} 3x+3, & x \leq 0 \\ 1 + \frac{4}{x}, & 0 < x \leq 2 \\ \frac{3x^2-6x}{2x-4}, & x > 2. \end{cases}$$

Strategy:

Example (continued): For the function $f(x)$ defined above, find:

(a) $\lim_{x \to 0^-} f(x)$ (b) $\lim_{x \to 0^+} f(x)$ (c) $\lim_{x \to 0} f(x)$ (d) $f(0)$

(e) $\lim_{x \to 2^-} f(x)$ (f) $\lim_{x \to 2^+} f(x)$ (g) $\lim_{x \to 2} f(x)$ (h) $f(2)$