MA 16010 Lesson 34: Numerical integration

Sometimes it is not practical/possible to evaluate integrals "analytically".

Examples:

$$\int \frac{\ln(1+t)}{(1+t)^2}\,\mathrm{d}t$$

$$\int e^{x^2}\,\mathrm{d}x$$

$\rightsquigarrow$ in practice, one often uses **numerical methods/approximations** to evaluate definite integrals.

Numerical method that we have seen already is the method of left/right Remann sums:

An improvement upon this is the **Trapezoidal Rule**:

Recall: How to compute the area of a trapezoid:

In the case of trapezoids from the previous picture, we get:

Altogether, the approximation of the integral is:

Example:

(A) Using the Trapezoidal Rule with 4 trapezoids, approximate the integral $\displaystyle\int_2^{10} (x^2 - 1)\,\mathrm{d}x$.

(B) Compute the integral precisely and compare:

Exercise: Using the Trapezoidal Rule with $n = 3$, approximate the integral $\displaystyle\int_0^9 (e^{x^2} - 1)\,\mathrm{d}x$.

Exercise: Using the Trapezoidal Rule with $n = 5$, approximate the area of the shaded region below.