MA 16010 Lesson 32: Fundamental Theorem of Calculus

Recall: If $y = f(x)$ is a function, we consider

1) the **definite integral** $\int_a^b f(x)\,\mathrm{d}x$:

– the result is

2) the **indefinite integral** $\int f(x)\,\mathrm{d}x$:

– the result is

Fundamental Theorem of Calculus relates the two integrals:

If ($f(x)$ is continuous on $[a, b]$ and) $\int f(x)\,\mathrm{d}x = F(x) + C$, then

$$\int_a^b f(x)\,\mathrm{d}x =$$

$\rightsquigarrow$ it gives a practical method to compute definite integrals.

Example. Let us compute $\int_1^3 (2x^3 + 3)\,\mathrm{d}x$:

Exercise: Compute the following definite integrals.

(a) $\displaystyle\int_1^4 \frac{x^2+\sqrt[3]{x^2}}{\sqrt[3]{x}}\,\mathrm{d}x$:

(b) $\displaystyle\int_0^5 (3e^x-8)\,\mathrm{d}x$:

(c) $\displaystyle\int_2^3 \frac{x+1}{x^2}\,\mathrm{d}x$:

Exercise: Find the area of the region enclosed by the curves given by the equations

$$y = 2\sin(x), \qquad y = 0, \qquad x = \frac{\pi}{4}, \qquad x = \frac{\pi}{2}.$$