MA 16010 Lesson 29: Area and Riemann Sums

Sigma notation. We use "Σ" to write sums of bunch of terms succintly.

For example, $\displaystyle\sum_{i=1}^{4} i^2 =$

Exercise: Evaluate

$$\sum_{i=2}^{5}(-1)^i(i-1)$$

$$\sum_{i=0}^{4}\frac{\sqrt{i}}{i+1}$$

Exercise: Use the Σ–notation to write down the sum

$$(\sqrt{3}-2)^2+(\sqrt{4}-3)^2+(\sqrt{5}-4)^2+\cdots+(\sqrt{n+2}-n-1)^2$$

$$=\sum_{i=1}^{n}$$

Area under the curve. For a function $y = f(x)$, we want to compute/ /estimate the **(signed) area under the curve** over a given interval $[a, b]$:

To approximate the area, we use **Riemann sums** =

Let's say we use n such rectangles ($n =$ in the picture above).

The base of each one has **length** $\Delta x =$ ________________

The **height** of each rectangle is:

For the **left Riemann sums**, it is ________________________________.

For the **right Riemann sums**, it is ________________________________.

The **area of one rectangle** is therefore ______________________________,

and the approximation of the **overall area** therefore is:

(Left R.S.) **(Right R.S.)**

Exercise: Use the left and right Riemann sums with 4 rectangles to estimate the (signed) area under the curve of

$$y = \sqrt[3]{x+2}$$

on the interval $[1, 9]$. (Round your answers to two decimal places.)

Exercise: Use the left and right Riemann sums with 100 rectangles to estimate the (signed) area under the curve of

$$y = e^x + 1$$

on the interval $[0, 10]$. (Write answers with the sigma notation.)