MA 16010 Lesson 21: Limits at ∞

Recall: The expression

$$\lim_{x \to c} f(x) = \infty$$

has the meaning:

"As x ____________________________, the value $f(x)$ ____________________________."

Example: $\lim_{x \to 0} \frac{1}{x^2} =$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$				—			

Now we switch it around: The expression

$$\lim_{x \to \infty} f(x) = c$$

has the meaning:

"As x ____________________________, the value $f(x)$ ____________________________."

Example: $\lim_{x \to \infty} \frac{1}{\sqrt{x}} =$

x	100	10000	1000000	100000000	...
$f(x)$					...

Similarly:

- We may also consider $\lim_{x \to -\infty} f(x)$:
- It may happen that $\lim_{x \to \infty} f(x) = \infty$, $\lim_{x \to \infty} f(x) = -\infty$ etc.

Limits at $\pm\infty$ of rational functions:

- We may use the same computational fules for limits as before:

$$\lim_{x\to\infty}\big(f(x)+g(x)\big)=\big(\lim_{x\to\infty}f(x)\big)+\big(\lim_{x\to\infty}g(x)\big),$$

$$\lim_{x\to\infty}\big(f(x)/g(x)\big)=\big(\lim_{x\to\infty}f(x)\big)/\big(\lim_{x\to\infty}g(x)\big),\quad \text{etc.}$$

- Useful observation: if c is a constant and a is a positive exponent, then

$$\lim_{x\to\infty}\frac{c}{x^a}=0,$$

$$\lim_{x\to-\infty}\frac{c}{x^a}=0,\quad \text{if it makes sense, e.g. if } a \text{ is positive}$$

Example:

$$\lim_{x\to-\infty}\left(\frac{3}{x}+\frac{x}{5}\right)=$$

Example:

$$\lim_{x\to\infty}\frac{3x^3+7x^2-10x+5}{x^2+8x-6}=$$

General rule:

Example:

$$\lim_{x\to-\infty}\frac{5x^2+3-2x^4}{8x-3x^4-1}=$$

Asymptotes. An asymptote of $f(x)$ is a line such that the graph of $f(x)$ tends to this line. Asymptotes can be

1. **Vertical:** $x = c$ is an asymptote for $f(x)$ if:

   in practice:

2. **Horizontal:** $y = c$ is an asymptote for $f(x)$ if:

3. **Slant:** $y = ax + b$ is an asymptote for $f(x)$ if:

   in practice:

Example: Find horizontal, vertical, slant asymptotes of

$$f(x) = \frac{2x^2 - 7x - 6}{3x^2 - 12} \quad .$$

Example: Find horizontal, vertical, slant asymptotes of

$$f(x) = \frac{x^3 - 7x - 6}{x^2 + x - 2} \quad .$$