MA 16010 Lesson 19: Concavity, inflection pts, 2nd derivative test

Recall:

(A) If $f'(x) > 0$ on an interval I, then f is ____________________ in I.

(B) If $f'(x) < 0$ on an interval I, then f is ____________________ in I.

Now let's go one step further:

(A') $f''(x) > 0$ on $I \Rightarrow$ ________________; then f is ________________ on I.

(B') $f''(x) < 0$ on $I \Rightarrow$ ________________; then f is ________________ on I.

Example: Find the largest intervals where $f(x) = x^3 - 3x^2 + 7x + 1$ is concave up and concave down.

A point (x, y) where $y = f(x)$ changes from concave up to concave down or vice versa is called ______________________________.

To find such points is to find ______________________________ of $f'(x)$!

Exercise: Find the largest intervals on which the function

$$f(x) = \frac{x^4}{3} + \frac{2}{3}x^3 - 4x^2 + x + 1$$

is concave up or concave down, and find the inflection points.

Summary - inflection points.

1.

2.

3.

Exercise: Find the largest intervals on which the function

$$f(x) = 5\ln(x^2 + 4)$$

is: (a) concave up or concave down, and find the inflection points.
(b) concave up and increasing (at the same time).

We may use concavity in finding relative extrema. If x is a point of:

(a) rel. max., then f is typically __________________, so we expect _________.

(b) rel. min., then f is typically __________________, so we expect _________.

Second derivative test: Let x be a critical point of $y = f(x)$.

1. If $f''(x) < 0$, then ____.
2. If $f''(x) > 0$, then ____.
3. In other cases, the test is inconclusive!

Exercise: Find the rel. extrema of $f(x) = \frac{2}{3}x^3 - x^2 - 12x + 5$.