MA 16010 Lesson 18: Incrasing & decreasing, first derivative test

Observation: Recall that the derivative of a function $y = f(x)$ has the meaning of *rate of change of f*. Therefore:

(A) If $f'(x) > 0$ on an interval I, then f is ____________________ in I.

(B) If $f'(x) < 0$ on an interval I, then f is ____________________ in I.

Application for relative extrema:

How to tell if a critical point is rel. maximum/rel. minimum?

- If c is the point of rel. maximum of f, then f is ______________________

  on some interval (a, c), ______________________ on some interval (c, b).

- If c is the point of rel. minimum of f, then f is ______________________

  on some interval (a, c), ______________________ on some interval (c, b).

Idea: Based on where $f'(x) < 0$ and where $f'(x) > 0$, determine which type of rel. extreme we are dealing with.

First derivative test: Given a critical point c of $f(x)$:

if ...	then ...
$f'(x) > 0$ on the left, $f'(x) < 0$ on the right,	____________________________ at c
$f'(x) > 0$ on the left, $f'(x) > 0$ on the right,	____________________________ at c
$f'(x) > 0$ on the left, $f'(x) > 0$ on the right,	____________________________ at c
$f'(x) < 0$ on the left, $f'(x) < 0$ on the right,	____________________________ at c

Strategy for relative extrema:

1.

2.

3.

Exercise: Find the rel. extrema of $f(x) = -2x^3 + 3x^2 + 12x + 5$.

Exercise: The derivative of a function $f(x)$ is $f'(x) = e^{3x}(x^3 + x^2 - 6x)$. Find the points of relative minima and maxima of $f(x)$.

Exercise: The critical points of $f(x) = 2\cos(2x) + 2x$ on $(0, 2\pi)$ are:

$$x = \frac{\pi}{12}\,, \quad x = \frac{5\pi}{12}\,, \quad x = \frac{13\pi}{12}\,, \quad x = \frac{17\pi}{12}\,.$$

Find the x–values in $(0, 2\pi)$ at which $f(x)$ has a relative maximum.