MA 16010 Lesson 17: Relative extrema, critical numbers

Given a function $y = f(x)$, we are often interested in its **maximal value** (e.g. "maximize profit") or its **minimal value** ("minimize costs"), if such values exist.

Today: We focus on **relative** maxima/minima.

- For a function $y = f(x)$ and a number c, we say that c **is the point of rel. maximum of** f / $f(c)$ **is a relative maximum** if:

Examples:

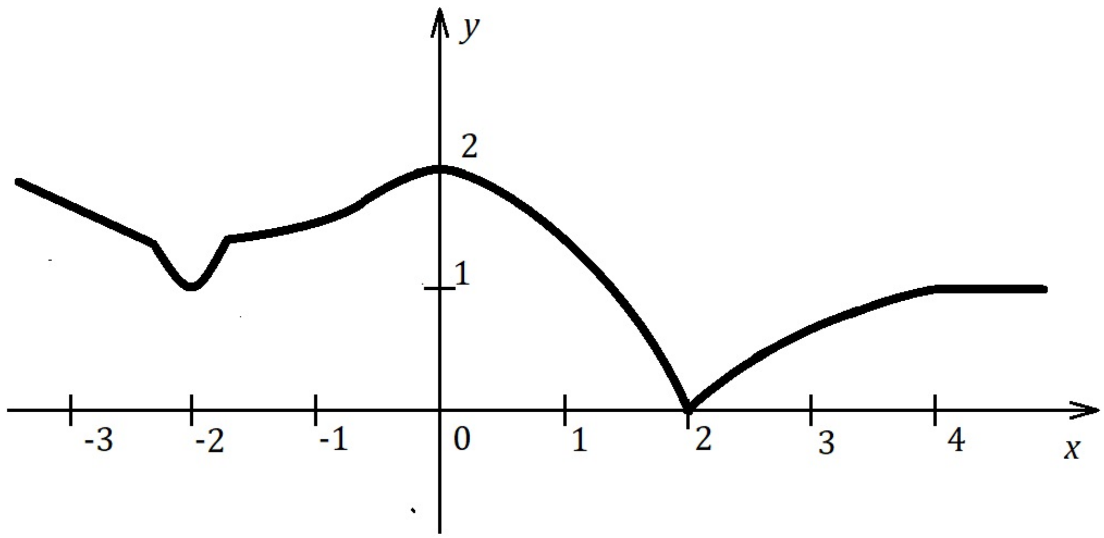
- For a function $y = f(x)$ and a number c, we say that c **is the point of rel. minimum of** f / $f(c)$ **is a relative minimum** if:

Examples:

- A number c is a **critical number** (critical point) of $y = f(x)$ if:

Examples:

Exercise: Find all relative extrema c, and describe $f'(c)$ at these points.



How to find relative extrema "analytically"?

Key observation: Relative minima, maxima are critical points $\rightarrow$ we find the critical points instead.

(**Warning:**)

How to find critical points:

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Exercise: Find the critical numbers for the following functions.

(a) $y = x^3 - 24x + 15$:

(b) $y = 2x^3 + 6x^2 + 6x + 1$:

(c) $y = x^4 - 4x^3 + 4x^2 - 5$:

Exercise: Find the critical numbers for the following functions.

(a) $y = x^2 - \frac{3}{x^2}$:

(b) $y = 3x^3 e^{2x+1}$:

(c) $y = \sin(2x) - 4x$, only in the interval $(0, \pi)$: