MA 16010 Lesson 14: Implicit Differentiation

Explicit vs. Implicit functions. A function in **explicit form** is what we considered so far. It is given by an equation of the form:

A function in **implicit form** is given by a more general equation involving x and y. We still think of $y = y(x)$ as a function of x.

Example: The function $y(x)$ given by the equation

$$x + 3y = 6$$

is in implicit form. The explicit form of the function is:

Example: The function $y(x)$ given by the equation

$$x^2 + y^2 = 4$$

is in implicit form. The explicit form of the function is:

either ____________________, or ____________________ (two functions!)

Implicit differentiation. Sometimes it is not easy/possible to find explicit form out of implicit one, but we can still take the derivative $\frac{\mathrm{d}}{\mathrm{d}x}$.

Idea: Differentiate both sides of the equation with respect to x. Treat y as a function of x, and use the chain rule wherevew appropriate, i.e.

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big[h(y)\Big] =$$

In the end, solve for y'.

Exercise: Using implicit differentiation, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when

$$4x^3 + 2xy^2 = 3y^3 - 7yx^2 \quad .$$

Exercise: Using implicit differentiation, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when

$$\cos(3x + 2y) = 5x^2y \quad .$$

Exercise: Using implicit differentiation, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when

$$3\cot\left(\frac{x}{y}\right) = 5x \quad .$$

Exercise: Find the slope of the tangent line to $\quad 3x^2 + 2y^2 = 14 \quad$ at $(2, 1)$.

Exercise: Find the equation of the tangent line to $6\sqrt{x} + 4\sqrt{y} = 5$ at $(x, y) = (1/4, 1/4)$.