MA 16010 Lesson 11: Chain rule I

Recall (composition of functions): Given two functions $f(x)$ and $g(x)$, their composition is the function $y =$ ________________.

Question for today: How to compute the derivative of a composite function in terms of the original functions?

Example: Compute the derivative of $h(x) = (x + \sin(x))^3$.

We have $h(x) = f(g(x))$ where $f(x) =$ _________ (so $f'(x) =$ _________),

and $g(x) =$ _________ (so $g'(x) =$ _________).

Using product rule (slow, complicated):

Chain rule: $\frac{\mathrm{d}}{\mathrm{d}x}\big[f(g(x))\big] =$

Exercise: Compute $y'(x)$ when $y = (x^{100} + 4)^{1000}$.

Another way to remember the chain rule:
Consider functions $y = f(u)$ and $u = g(x)$. We may consider $y = f(g(x))$ to be the composite function. Then

$$\frac{\mathrm{d}y}{\mathrm{d}x} =$$

Exercise: Use the chain rule to compute $h'(x)$ when:

$h(x) = (\cos(x) + \tan(x))^{-5}$:

$h(x) = \sqrt[3]{x^7 + 8}$:

$h(x) = \left(\frac{3x}{x+5}\right)^8$:

Exercise: Compute $h'(\ln(\pi))$ for

$$h(x) = \cos(e^x + \pi/2).$$

Exercise: Compute the derivative $h'(x)$ for

$$h(x) = e^{200x}$$

using the chain rule in two different ways.