MA 16010 Quiz 10 (Lessons 24 – 26)

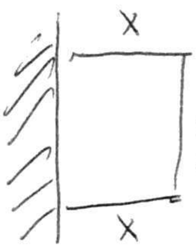
Write your name, section number (399 for 8:30, 418 for 9:30), and quiz number on the top of your quiz, **front and back.** You may use a one–line calculator.

1. We are putting up a fence around three sides of a rectangular garden (the fourth side is next to a wall). The area of the garden should be $72\,\text{m}^2$. What dimensions should the garden have if the fence is to be the shortest possible?

2. For a cylinder with a surface area of 50 ft^2, what is the maximum volume that it can have? Round your answer to three decimal places.

   (**Recall** that the volume of a cylinder is $\pi r^2 h$ and the surface area is $2\pi r h + 2\pi r^2$ where r is the radius and h is the height.)

$\boxed{1.}$



Want: minimize length of fence $\ell = 2x + y$

Constraint: $A = xy = 72 \rightsquigarrow y = \frac{72}{x}$

$\rightsquigarrow$ obj. function $\ell = 2x + \frac{72}{x}$

$\ell' = 2 - \frac{72}{x^2} = 0$

$\frac{72}{x^2} = 2$

$2x^2 = 72 \rightarrow x^2 = 36$

$x = \pm 6$

$x = 6\,\text{m}$

$y = \frac{72}{6} = 12\,\text{m}$

$\boxed{2.}$

Want: maximize volume $V = \pi r^2 h$

Constraint: surface area $= 50\,\text{ft}^2$ ... $2\pi r h + 2\pi r^2 = 50$

$h = \frac{50 - 2\pi r^2}{2\pi r}$

$\rightsquigarrow$ obj. function

$\rightarrow V = \cancel{\pi} r^{\cancel{2}} \cdot \frac{50 - 2\pi r^2}{2\cancel{\pi}\cancel{r}} = r \cdot \frac{50 - 2\pi r^2}{2} = 25r - \pi r^3$

$V' = 25 - 3\pi r^2 = 0$

$3\pi r^2 = 25$

$r^2 = \frac{25}{3\pi}$

$r = \sqrt{\frac{25}{3\pi}} = \frac{5}{\sqrt{3\pi}}$

Max volume $= 25 \cdot \left(\frac{5}{\sqrt{3\pi}}\right) - \pi \cdot \left(\frac{5}{\sqrt{3\pi}}\right)^3\ \text{ft}^3$

$\left(= \frac{250}{3\sqrt{3\pi}}\ \text{ft}^3\right) \approx 27.145\ \text{ft}^3$