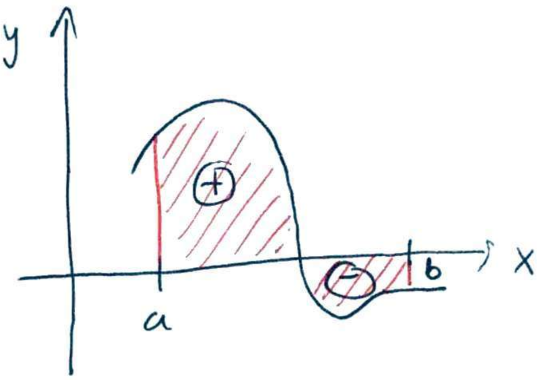
MA 16010 Lesson 31: Definite Integrals II

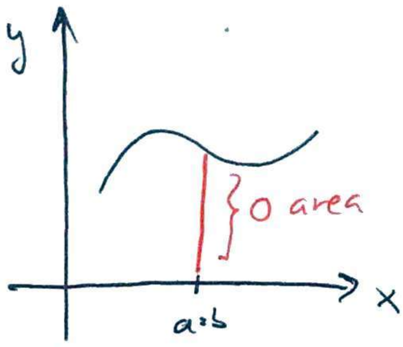
Recall: The geometric meaning of the definite integral $\int_a^b f(x)\,dx$ is:

the (signed) area under the curve $y=f(x)$, above $[a,b]$:

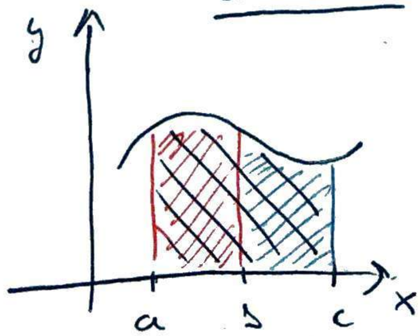


"Algebraic rules" for definite integrals. Assume $a \le b \le c$.

1. $\displaystyle\int_a^a f(x)\,dx = 0$

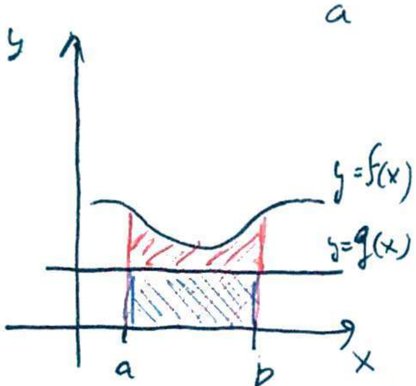


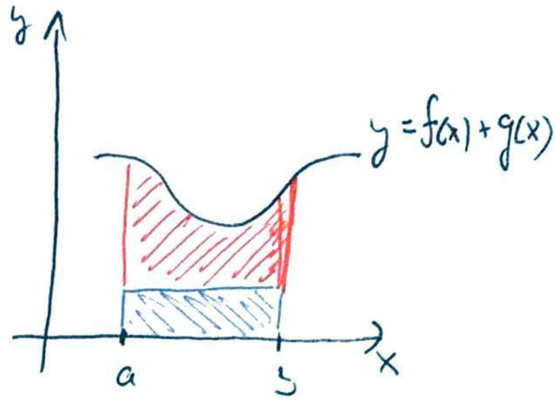
2. $\displaystyle\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$



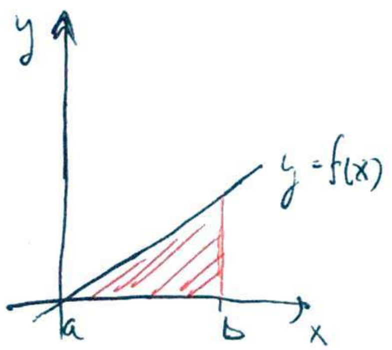
3. $\displaystyle\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$ (*"convention"*)

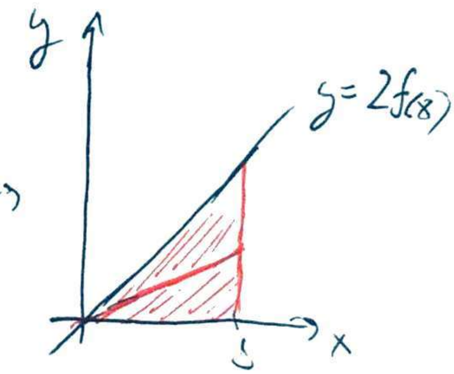
4. $\displaystyle\int_a^b (f(x) \pm g(x))\,\mathrm{d}x = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$





5. $\displaystyle\int_a^b (k \cdot f(x))\,\mathrm{d}x = k \cdot \int_a^b f(x)\,dx$ (k a constant)





Exercise: Assuming that $\displaystyle\int_2^4 6x^2\,\mathrm{d}x = 112$,

(a) find $\displaystyle\int_4^2 6x^2\,\mathrm{d}x$: $\displaystyle\int_4^2 6x^2\,dx = -\int_2^4 6x^2\,dx = -\underline{\underline{112}}$

[rule 3.]

(b) find $\displaystyle\int_4^2 15x^2\,\mathrm{d}x$: $\displaystyle\int_4^2 15x^2\,dx = \int_4^2 \frac{5}{2}\cdot 6x^2\,dx = \frac{5}{2}\int_4^2 6x^2\,dx =$

[rule 5.]

$= \frac{5}{2}\cdot(-112) = 5\cdot(-56) = \underline{\underline{-280}}$

Exercise: Given that $\int_0^3 x^2\,\mathrm{d}x = 9$, $\int_3^6 x^2\,\mathrm{d}x = 63$ and $\int_0^6 x^3\,\mathrm{d}x = 324$,

find $\int_0^6 (4x^2 - x^3)\,\mathrm{d}x$. [rules 4., 5.]

$$\int_0^6 (4x^2 - x^3)\,dx = 4\int_0^6 x^2\,dx - \int_0^6 x^3\,dx = 4\cdot\int_0^6 x^2\,dx - 324$$

Need $\int_0^6 x^2\,dx$, which is $\int_0^3 x^2\,dx + \int_3^6 x^2\,dx = 9 + 63 = 72$ [rule 1.]

$$\rightsquigarrow \int_0^6 (4x^2 - x^3)\,dx = 4\cdot 72 - 324 = 288 - 324 = \underline{\underline{-36}}$$

Exercise: Given that $\int_{-4}^{7} f(t)\mathrm{d}t = 31$, $\int_{-4}^{-1} f(t)\mathrm{d}t = 8$ and $\int_{-1}^{7} g(t)\mathrm{d}t = 11$,

find $\int_{-1}^{7} (g(t) - 2f(t))\,\mathrm{d}t$.

$$\int_{-1}^{7} (g(t) - 2f(t))\,dt = \int_{-1}^{7} g(t)\,dt - 2\cdot\int_{-1}^{7} f(t)\,dt = 11 - 2\cdot\int_{-1}^{7} f(t)\,dt$$

$\rightsquigarrow$ need $\int_{-1}^{7} f(t)\,dt$. Know: $\underbrace{\int_{-4}^{-1} f(t)\,dt}_{8} + \int_{-1}^{7} f(t)\,dt = \underbrace{\int_{-4}^{7} f(t)\,dt}_{31}$

$$\rightarrow \int_{-1}^{7} f(t)\,dt = 31 - 8 = \underline{23}$$

so $$\int_{-1}^{7} (g(t) - 2f(t))\,dt = 11 - 2\cdot 23 = 11 - 46 = \underline{\underline{-35}}$$

Exercise: Given that $\int_a^b f(x)\mathrm{d}x = 14$ and $\int_a^c f(x)\mathrm{d}x = 2 \cdot \int_c^b f(x)\mathrm{d}x$,

find $\int_c^b f(x)\mathrm{d}x$.

Know: 1) $\int_a^c f(x)dx + \int_c^b f(x)dx = \underbrace{\int_a^b f(x)dx}_{14} = 14$

2) $\int_a^c f(x)dx = 2 \cdot \int_c^b f(x)dx$

Plug in 2) into 1):

$$2 \cdot \int_c^b f(x)dx + \int_c^b f(x)dx = 14$$

$$3 \cdot \int_c^b f(x)dx = 14$$

$$\int_c^b f(x)dx = \frac{14}{3}$$

Bonus question: what is $\int_a^c f(x)dx$?

[it is $2 \cdot \int_c^b f(x)dx = \frac{28}{3}$]