MA 16010 Lesson 21: Properties of f from f' (graphically)
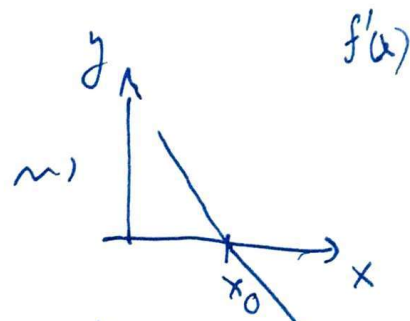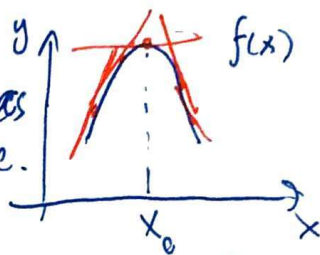
Recall: We may find the following information about a function $y = f(x)$ in terms of $f'(x)$:

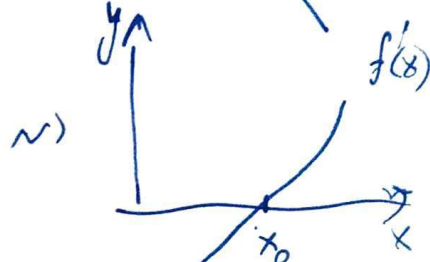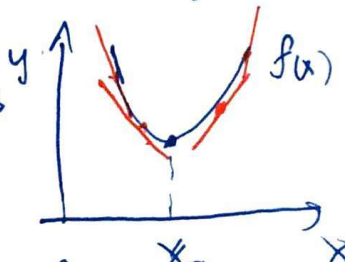**Critical numbers of f:** x s.uch that $f'(x) = 0$

Where f is increasing: = where $f'(x) > 0$

Where f is decreasing: = where $f'(x) < 0$

Point of relative maximum of f:
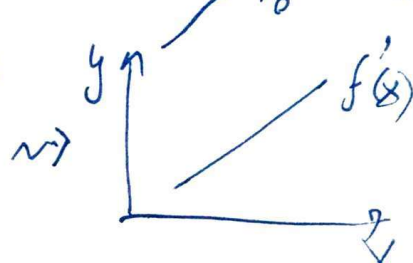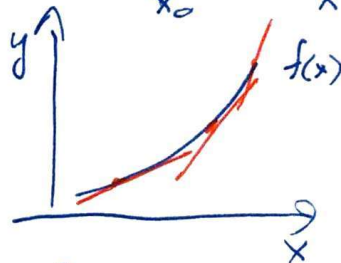= the point where $f'(x)$ switches from being positive to being negative.

Point of relative minimum of f:
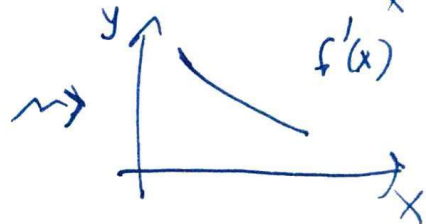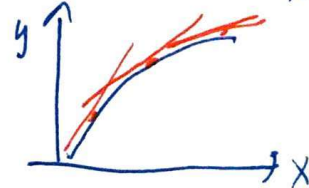= the point where $f'(x)$ switches from negative to positive

Where f is concave up:
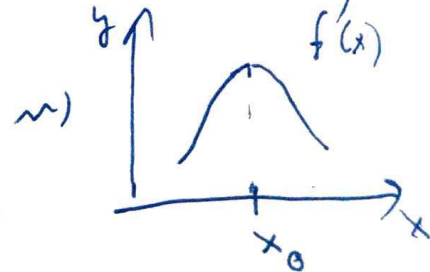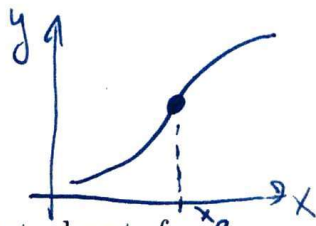$(= $ where $f''(x) > 0)$
= where $f'(x)$ is increasing

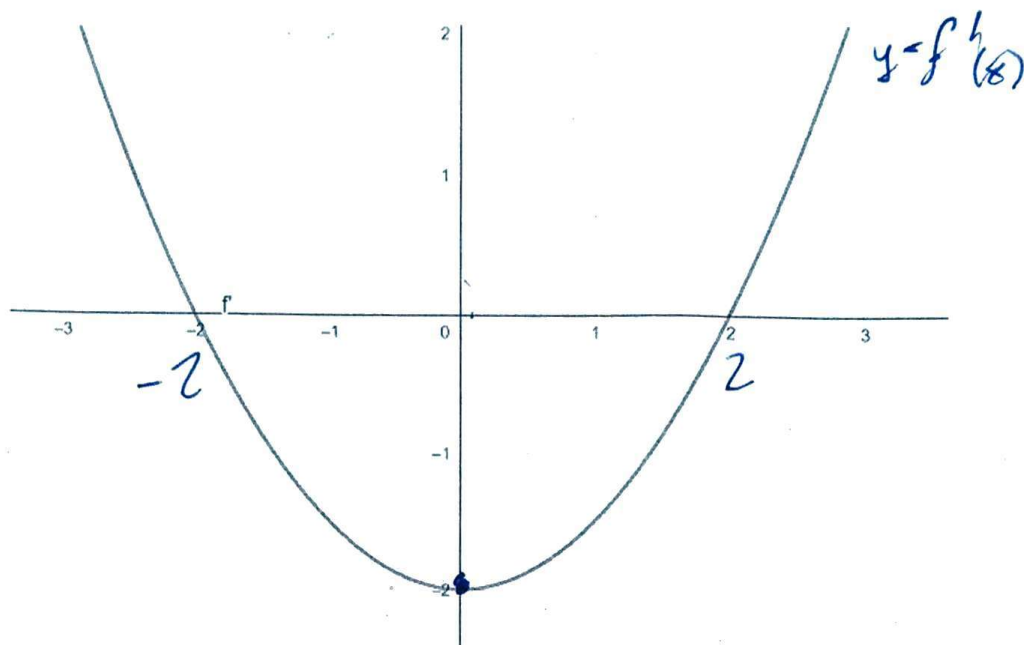**Where f is concave down:**
= where $f'(x)$ is decreasing

Where f has inflection point (x–coordinate):
= the points of rel. extrema of $f'(x)$

$\rightarrow$ From the graph of f', we can tell a lot about f.

Exercise: Given the graph of $f'(x)$ below, find the following about $f(x)$:

$y = f'(x)$

Critical numbers: $x = -2, \; x = 2$

Increasing intervals: $(-\infty, -2), \; (2, \infty)$

Decreasing intervals: $(-2, 2)$
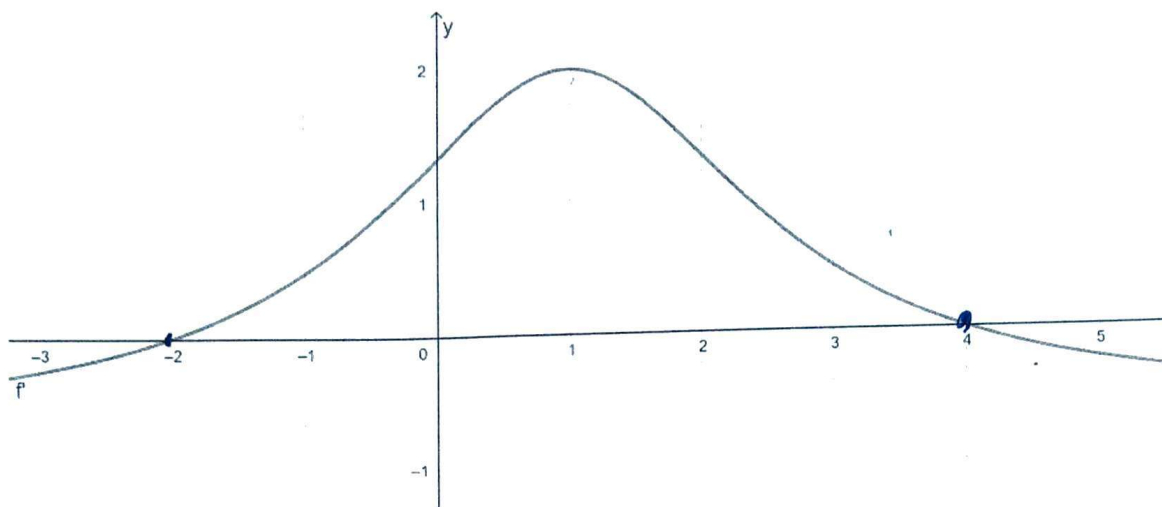
Relative maxima of f occur at $x = -2$

Relative minima of f occur at $x = 2$

Concave up intervals: $(0, \infty)$

Concave down intervals: $(-\infty, 0)$

Inflection points (x–coordinate): $x = 0$

Exercise: Given the graph of $f'(x)$ below, find the following about $f(x)$:

Critical numbers: $x = -2,\ x = 4$

Increasing intervals: $(-2, 4)$

Decreasing intervals: $(-\infty, -2),\ (4, \infty)$
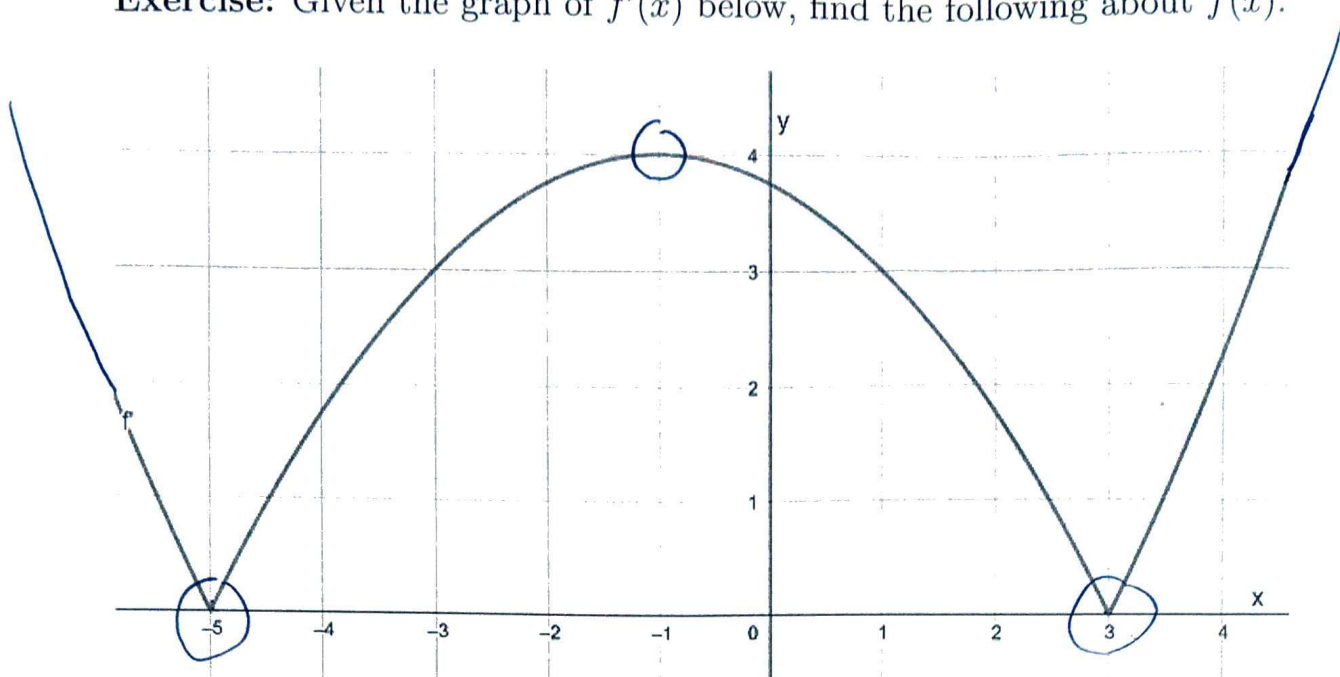
Relative maxima of f occur at $x = 4$

Relative minima of f occur at $x = -2$

Concave up intervals: $(-\infty, 1)$

Concave down intervals: $(1, \infty)$

Inflection points (x–coordinate): $x = 1$

Exercise: Given the graph of $f'(x)$ below, find the following about $f(x)$:

Critical numbers: $x = -5$, $x = 3$

Increasing intervals: $(-\infty, -5), (-5, 3), (3, \infty)$ $\sim$ $(-\infty, \infty)$
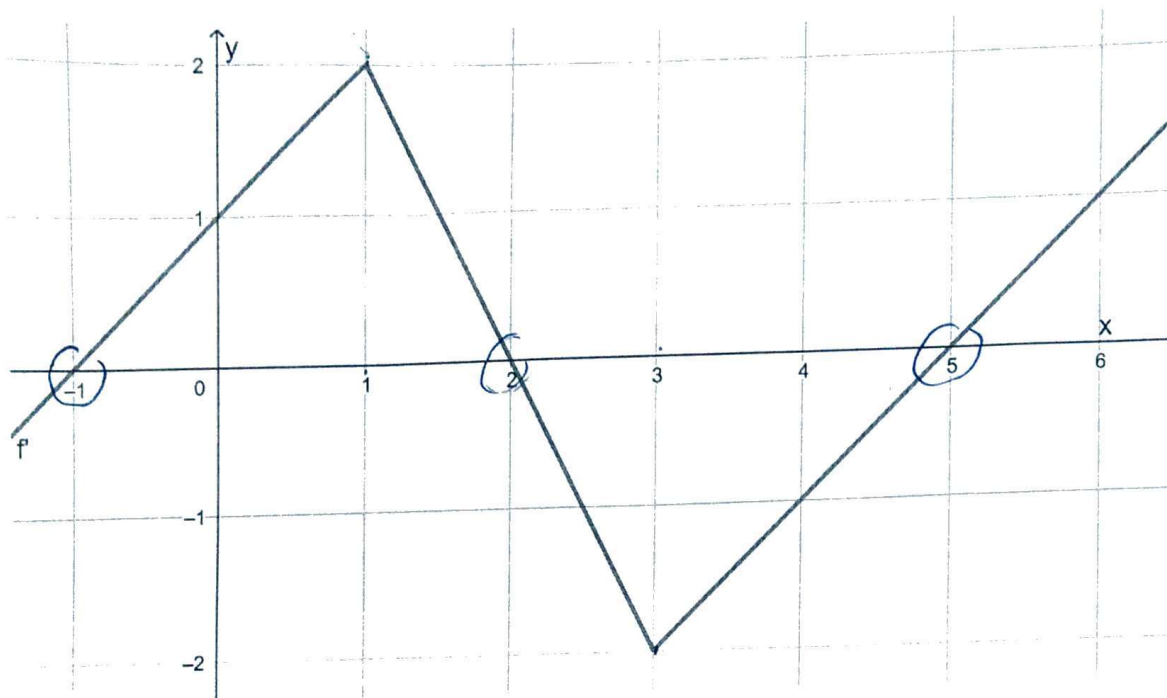
Decreasing intervals: none

Relative maxima of f occur at $x=$ none

Relative minima of f occur at $x=$ none

Concave up intervals: $(-5, -1), (3, \infty)$

Concave down intervals: $(-\infty, -5), (-1, 3)$

Inflection points (x–coordinate): $x = -5$, $x = -1$, $x = 3$

Exercise: Given the graph of $f'(x)$ below, find the following about $f(x)$:

Critical numbers: $x = -1$, $x = 2$, $x = 5$

Increasing intervals: $(-1, 2)$, $(5, \infty)$

Decreasing intervals: $(-\infty, -1)$, $(2, 5)$

Relative maxima of f occur at $x = 2$

Relative minima of f occur at $x = -1$, $x = 5$

Concave up intervals: $(-\infty, 1)$, $(3, \infty)$

Concave down intervals: $(1, 3)$

Inflection points (x-coordinate): $x = 1$, 3