MA 16010 Lesson 13: Higher order derivatives

Given a function $f(x)$, its derivative $f'(x)$ is yet another function. Taking the derivative of *this* function yields $f''(x)$, the second derivative of f.

other notation: $f^{(2)}(x)$, $y''(x)$, $\frac{d^2f}{dx^2}\left(= \frac{d}{dx}\left(\frac{d}{dx}(f)\right)\right)$, $\frac{d^2y}{dx^2}$, ...

We can continue in this manner, to obtain other *higher order derivatives*:

- $f'(x)$ 1st derivative
- $f''(x)$ 2nd derivative
- $f^{(3)}(x)$ 3rd derivative
- $f^{(4)}(x) = \frac{d^4f}{dx^4}$ 4th derivative, ...

take derivative

take derivative

take derivative

...

Exercise: Compute $f''(x)$ for $f(x) = x^4 + 3x^2 + 1$.

$$f'(x) = 4x^3 + 6x$$

$$\underline{\underline{f''(x) = 12x^2 + 6}}$$

Exercise: Compute $f''(-1)$ for $f(x) = x^2 e^x$.

$$f'(x) = 2x \cdot e^x + x^2 \cdot e^x = (2x + x^2)e^x$$

$$f''(x) = 2 \cdot e^x + 2x e^x + 2x e^x + x^2 e^x$$

$$= (2 + 2x + 2x + x^2)e^x = (x^2 + 4x + 2)e^x$$

$$f''(-1) = ((-1)^2 + 4 \cdot (-1) + 2) \cdot e^{-1}$$

$$= (-1) \cdot e^{-1} = -\frac{1}{e}$$

Exercise: Compute the third derivative of $f(x) = \sin(3x)$.

$$f'(x) = \cos(3x) \cdot 3 = 3\cos(3x)$$

$$f''(x) = 3 \cdot (-\sin(3x)) \cdot 3 = -9\sin(3x)$$

$$f'''(x) = -9\cos(3x) \cdot 3 = \underline{\underline{-27\cos(3x)}}$$

Exercise: Compute the second derivative of $y = \ln(x^2 + 1)$.

chain rule:
$f(u) = \ln(u)$
$f'(u) = \frac{1}{u}$
$g(x) = x^2+1$
$g'(x) = 2x$

$$y' = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$$

$$y'' = \frac{2(x^2+1) - (2x)\cdot(2x)}{(x^2+1)^2} =$$

$$= \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{2-2x^2}{(x^2+1)^2}$$

Application: motion. Recall that if $s(t)$ is the function of a position of an object depending on time, its rate of change $s'(t)$ is the velocity, $v(t)$.
Similarly, if one takes the rate of change of the velocity, $v'(t)$, one obtains the acceleration, $a(t)$.

Altogether, we have:

$$\text{acceleration} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{d}{dt}s\right) =$$

$$\boxed{a(t) = \frac{d^2s}{dt^2}}$$

Exercise: The position of a particle moving on a straight line is given by

$$s(t) = \sin(2t) + 7t^2 \qquad [m]$$

(in meters, where t is time in seconds). What is the acceleration of the particle at $t = 5$ seconds? Round your answer to 3 decimal places.

$a(t) = \frac{d^2 s}{dt^2}$; $s'(t) = \cos(2t) \cdot 2 + 14t \quad [= v(t)]$

$a(t) = s''(t) = 2(-\sin(2t)) \cdot 2 + 14 =$

$= 14 - 4\sin(2t) \quad [m/s^2]$

$a(5) = 14 - 4 \cdot \sin(10) \approx \underline{16.176 \; m/s^2}$

Exercise: The velocity of a particle moving on a straight line is given by

$$v(t) = 3t^2 - 3 \qquad [m/s]$$

(in meters per second, where t is time in seconds). What is the acceleration of the particle at $t = 2$ seconds?

$a(t) = \frac{dv}{dt} = 6t$

$\underline{a(2) = 12 \; m/s^2}$