Assigned: Friday Jan. 29, 2008
Due: Friday Feb. 5, 2008

MATH 442: Homework 3
Spring 2010

NOTE: For each homework assignment observe the following guidelines:

- Include a cover page.
- At the beginning of each problem, clear state the problem (with assumptions) and what method or approach you intend to use to solve the problem.
- Clear mark your answer.
- Always clearly label all plots (title, x-label, y-label, and legend).
- Use the `subplot` command from MATLAB when comparing 2 or more plots to make comparisons easier and to save paper.

1. Compute the following integrals where m and n are non-negative integers. Look out for special cases.

   (a) $\int_0^L \cos(n\pi x/L)\cos(m\pi x/L)\,dx$

   (b) $\int_0^L \cos(n\pi x/L)\sin(m\pi x/L)\,dx$

2. Consider the boundary value problem:

$$\begin{aligned} \text{PDE:} \quad & u_t = u_{xx}, \ (0 < x < 4) \\ \text{BCs:} \quad & u_x(0,t) = -2, \ u_x(4,t) = -2 \\ \text{ICs:} \quad & u(x,0) = \begin{cases} 0 & \text{if } 0 \le x \le 2 \\ 2x-4 & \text{if } 2 \le x \le 4. \end{cases} \end{aligned}$$

   (a) Find the steady-state solution. (**NOTE:** There are Neumann BCs at each end. Generally, this would suggest that there would *not* be a steady-state solution. In this case, however, the *same* value of the derivative is given at each end, meaning that heat leaves and enters at the same rate. You will find that the steady-state solution contains an arbitray constant. So how do you chose this constant? Since the net heat flux into the interval $0 \le x \le 4$ is 0, the total heat energy must not depend on time. Choose the constant in the steady-state solution so that the total energy as $t \to \infty$ is the same as the energy at time $t = 0$.)

   (b) Using separation of variables, find the solution $u(x,t)$ to this problem. In order to accomplish this do the following:

      - Write $u(x,t) = w(x) + v(x,t)$, where $w(x)$ is the particular solution (also the *steady-state* solution) and $v(x,t)$ is the homogeneous solution.
      - Write down the PDE and the BCs that $v(x,t)$ must satisfy.
      - Write down the solution for $v(x,t)$ using the separation of variables results we obtained in class.

- Choose the arbitrary constants in $v(x,t)$ so that $u(x,t)$ satisfies the initial condition.

(c) Plot $u(x,t)$ as a function of x for several values of t in order to see how the temperature profile evolves from the inital condition towards the steady-state solution. (**NOTE:** use the `subplot` command in MATLAB in order to save paper. In each `subplot` window plot $u(x,t)$ at a given instant in time.)

3. Using separation of variables solve the following BVP:

$$\begin{aligned} \text{PDE:}\quad & u_{xx} + u_{yy} = 0 \\ \text{BCs:}\quad & u(0,y) = 0 \\ & u(L,y) = 0 \\ & u(x,0) = 0 \\ & u(x,H) = f(x). \end{aligned}$$

4. Using separation of variables, solve Laplace's equation inside a 60° wedge of radius a, subject to the BCs:

$$\begin{aligned} u(r,0) &= 0 \\ u(r,\pi/3) &= 0 \\ u(a,\theta) &= f(\theta). \end{aligned}$$