

3. (5 points) Suppose that  $f$  and  $g$  are functions such that

$$f(g(x)) = x \quad (*)$$

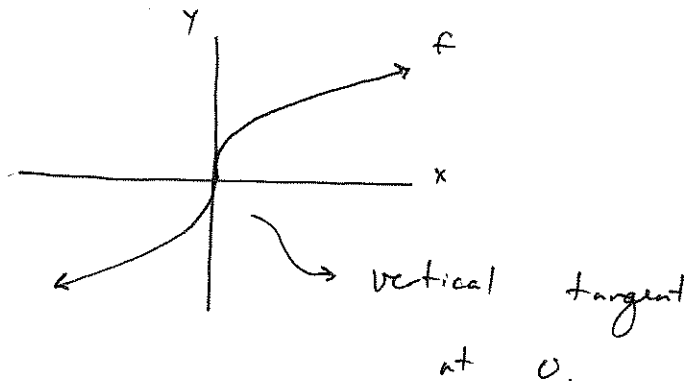
for every  $x$ , and that  $f'(x) = \frac{1}{x}$ . Prove that  $g'(x) = g(x)$  by differentiating (\*).

By the chain rule:

$$\begin{aligned} 1 &= \frac{d}{dx} x = \frac{d}{dx} f(g(x)) = f'(g(x)) g'(x) \\ &= \frac{1}{g(x)} g'(x) \end{aligned}$$

Multiply by  $g(x)$  to get  $g = g'$ .

4. (5 points) Draw the graph of a function  $f$  which is continuous on  $(-\infty, \infty)$ , differentiable on  $(-\infty, 0) \cup (0, \infty)$  (and *not* differentiable at 0), and  $f'(x) > 0$  wherever  $f$  is differentiable.



Example function:  $x^{1/3}$ .