Preface

In recent years there has been a resurgence of interest in representations of symmetric groups (as well as other Coxeter groups). This topic can be approached from three directions: by applying results from the general theory of group representations, by employing combinatorial techniques, or by using symmetric functions. The fact that this area is the confluence of several strains of mathematics makes it an exciting one in which to study and work. By the same token, it is more difficult to master.

The purpose of this monograph is to bring together, for the first time under one cover, many of the important results in this field. To make the work accessible to the widest possible audience, a minimal amount of prior knowledge is assumed. The only prerequisites are a familiarity with elementary group theory and linear algebra. All other results about representations, combinatorics, and symmetric functions are developed as they are needed. Hence this book could be read by a graduate student or even a very bright undergraduate. For researchers I have also included topics from recent journal articles and even material that has not yet been published.

Chapter 1 is an introduction to group representations, with special emphasis on the methods of use when working with the symmetric groups. Because of space limitations, many important topics that are not germane to the rest of the development are not covered. These subjects can be found in any of the standard texts on representation theory.

In Chapter 2, the results from the previous chapter are applied to the symmetric group itself and more highly specialized machinery is developed to handle this case. I have chosen to take the elegant approach afforded by the Specht modules rather than working with idempotents in the group algebra.

The third chapter focuses on combinatorics. It starts with the two famous formulae for the dimensions of the Specht modules: the Frame-Robinson-Thrall hook formula and the Frobenius-Young determinantal formula. The centerpiece is the Robinson-Schensted-Knuth algorithm, which allows us to describe some of the earlier theorems in purely combinatorial terms. A thorough discussion of Schützenberger's jeu de taquin and related matters is included.

Chapter 4 recasts much of the previous work in the language of symmet-

ric functions. Schur functions are introduced, first combinatorially as the generating functions for semistandard tableaux and then in terms of symmetric group characters. The chapter concludes with the famous Littlewood-Richardson and Murnaghan-Nakayama rules.

My debt to several other books will be evident. Much of Chapter 1 is based on Ledermann's exquisite text on group characters ["Introduction to Group Characters," Cambridge University Press, Cambridge, 1977]. Chapter 2 borrows heavily from the monograph of James ["The Representation Theory of Symmetric Groups," Lecture Notes in Math., Vol. 682, Springer-Verlag, New York, NY, 1978]. Whereas Chapter 4 is inspired by Macdonald's already classic book ["Symmetric Functions and Hall polynomials," Oxford University Press, Oxford, 1979].

Finally, the third chapter is a synthesis of material from the research literature.

There are numerous applications of representations of groups, and in particular of the symmetric group, to other areas. For example, they arise in physics, probability and statistics, topological graph theory, and the theory of partially ordered sets. However, to keep the length of this text reasonable, I have discussed only the connections with combinatorial algorithms.

This book grew out of a course that I taught while visiting the Université du Québec à Montréal during the fall of 1986. I would like to thank *l'équipe de combinatoire* for arranging my stay. I also presented this material in a class here at Michigan State University in the winter and spring of 1990. I thank my students in both courses for many helpful suggestions (and those at l'UQAM for tolerating my bad French). Francesco Brenti, Kathy Dempsey, Yoav Dvir, Kathy Jankoviak, and Scott Mathison have all pointed out ways in which the presentation could be improved. I also wish to express my appreciation of John Kimmel, Marlene Thom, and Linda Loba at Wadsworth and Brooks/Cole for their help during the preparation of the manuscript. Because I typeset this document myself, all errors can be blamed on my computer.

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