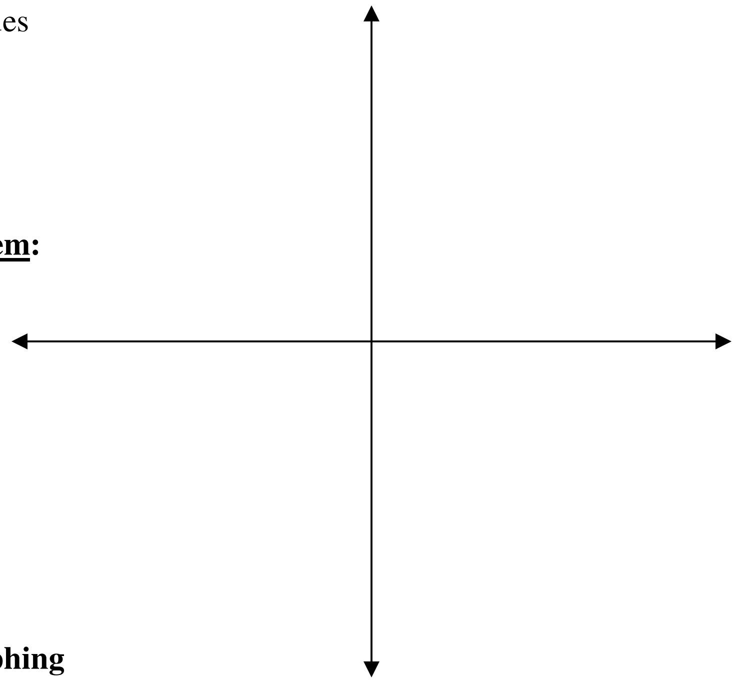
2.1 The Rectangular Coordinate System

In this section you will learn to:

- plot points in a rectangular coordinate system
- understand basic functions of the graphing calculator
- graph equations by generating a table of values
- graph equations using *x*- and *y*-intercepts



<u>Understanding the Rectangular Coordinate System</u>:

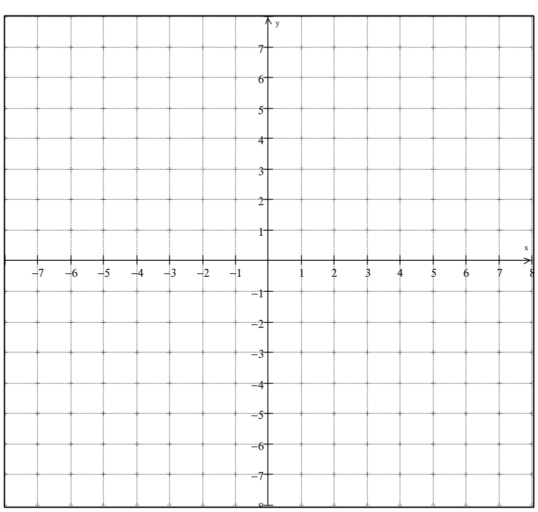
- points/ordered pairs
- origin
- *x*- and *y*-axes
- quadrants

For this course you must be able to use your graphing calculator to perform the following functions:

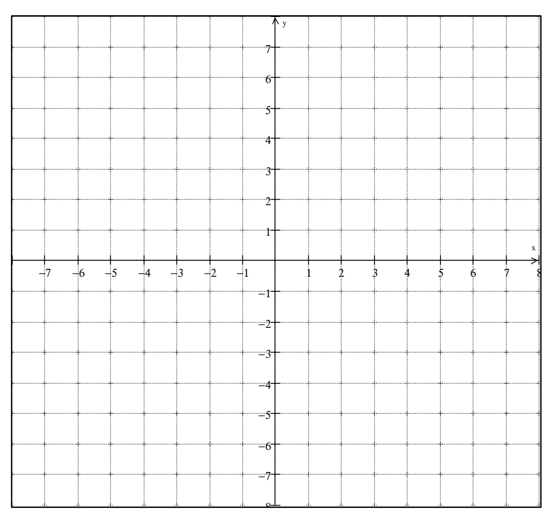
1. enter equations (You must be able to solve the equation for *y*.)
2. generate a table of values
3. determine an appropriate viewing rectangle (window)
4. graph equations using an appropriate window

Example 1: Generate a table of values to graph the equations below without using a calculator. Then check the table values and graph using a graphing calculator.

(a) $y = 2 - x^2$



(b) $y = |x| - 3$



The standard **viewing rectangle** or **viewing window** for most calculators is [-10, 10, 1] by [-10, 10, 1] or [minimum *x*-value, maximum *x*-value, *x*-axis scale] by [minimum *y*-value, maximum *y*-value, *y*-axis scale] determined by the *x*- and *y*-axes.

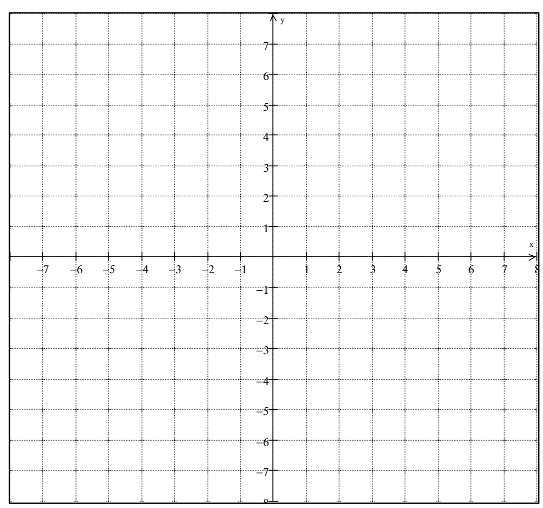
The viewing rectangle for the graph in Example 1 is ____.

Example 2: Draw a viewing rectangle to represent [-8, 10, 2] by [-10, 15, 5].

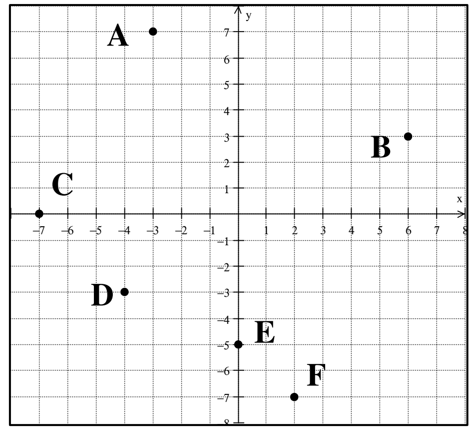
The ***x*-intercept** of a graph is the *x*-coordinate of a point where the graph intersects the *x*-axis.
To find the *x*-intercept:
1. Substitute 0 for *y*-value.
2. Solve for *x*.

The ***y*-intercept** of a graph is the *y*-coordinate of a point where the graph intersects the *y*-axis.
To find the *y*-intercept:
1. Substitute 0 for *x*-value.
2. Solve for *y*.

Example 3: Graph the equation $4x - 6y = 12$ using intercepts.



2.1 Homework Problems



1. Refer to the graph at right to determine the coordinates of points A – F.

2. Determine in which quadrant(s) or on which axis the point (x, y) must lie based on the following conditions:

   (a) $x > 0$ and $y < 0$ (b) $x < 0$ and $y < 0$

   (c) $xy > 0$ (d) $xy < 0$

   (e) $xy < 0$ and $y > 0$ (f) $x > 0$ and $y = 0$

   (g) $xy > 0$ and $y < 0$ (h) $y < 0$ and $x = 0$

3. Complete the table of values for $y = 2x^2 - 5$ to find coordinates (x, y).

x	-3	-2	-1	0	1	2	3
y							

4. Given the equation $y = 2 - |x|$, find the y-values for each of the ordered pairs:
   (-3,), (-2,), (-1,), (0,), (1,), (2,), (3,)

5. Find the *x*- and *y*-intercepts of the graphs for each of the equations.

   (a) $3x + 5y - 15 = 0$ (b) $2x - 3y = -24$

   (c) $5x + 3y + 20 = 0$ (d) $\frac{2}{3}x - \frac{1}{2}y + 1 = 0$

   (e) $3(y + 2) = 2x - 3$

6. A car purchased for $18,270 is expected to depreciate according to the formula $y = -1260x + 18270$, where *y* is the value after *x* years. When will the car no longer have any value?

2.1 Homework Answers: 1. A(-3, 7); B(6, 3); C(-7, 0); D(-4, -3); E(0, -5); F(2, -7) **2**. (a) IV; (b) III; (c) I or III; (d) II or IV; (e) II; (f) positive *x*-axis; (g) III; (h) negative *y*-axis **3.** (-3, 13); (-2, 3); (-1, -3); (0, -5); (1, -3); (2, 3); (3, 13) **4.** (-3, -1); (-2, 0); (-1, 1); (0, 2); (1,1); (2, 0); (3, -1)

5. (a) (5, 0) and (0, 3); (b) (-12, 0) and (0, 8); (c) (-4, 0) and $\left(0, -\frac{20}{3}\right)$; (d) $\left(-\frac{3}{2}, 0\right)$ and (0, 2); (e) $\left(\frac{9}{2}, 0\right)$ and (0, -3) **6.** in 14.5 years

2.2 Slope and Average Rate of Change

In this section you will learn to:

- find the slope of an oblique (slanted) line
- find the slope of horizontal and vertical lines
- find the average rate of change

Definition: The **slope** of the line through the distinct points (x_1, y_1) and (x_2, y_2) is

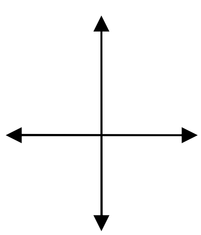
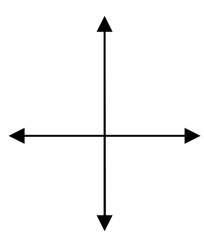
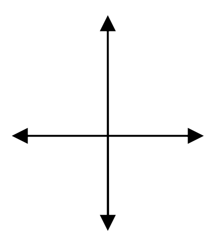
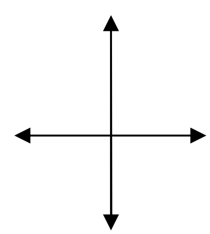
$$\frac{\textit{change in } y}{\textit{change in } x} \quad \text{or} \quad \frac{\textit{rise}}{\textit{run}} \quad \text{or} \quad \frac{\Delta y}{\Delta x} \quad \text{or} \quad \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad \frac{y_1 - y_2}{x_1 - x_2}, \quad \text{where } x_2 \neq x_1 .$$

Example 1: Find the slope of the line containing the following points:

(a) (-3, -5) and (-2, 6)

(b) $(-5, 6)$ and (8, 6)

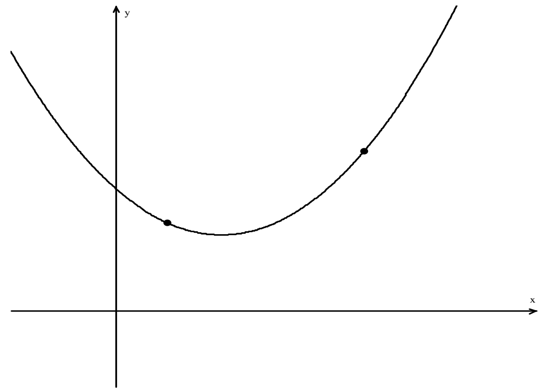
(c) $(6, -5)$ and (6, 8)

Positive Slope	**Negative Slope**	**Zero Slope**	**Undefined Slope**
m > 0	m < 0	m = 0	m is undefined
			

If a graph is not a straight line, the **average rate of change** between any two points is the slope of the line containing the two points. This line is called a secant line.

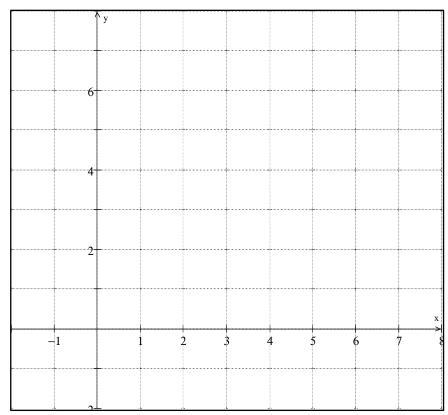
Let (x_1, y_1) and (x_2, y_2) be distinct points on a graph. The **average rate of change** from x_1 to x_2 is

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{where } x_2 \neq x_1$$

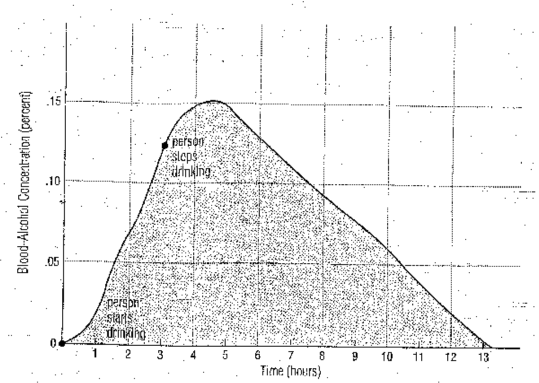


Example 2: The minimum wage in 1967 was \$1.40. The minimum wage in 2009 was \$7.25. Find the **average rate of change** in the minimum wage from 1967 to 2009. Round to nearest cent.

Example 3: Find the average rate of change on the graph of $y = (x-2)^2$ from $x_1 = 2$ to $x_2 = 3$.



Example 4: Refer to the graph below to find the average rate of change (ARC) of the blood alcohol level 6 to 10 hours after drinking. What does this represent?



2.2 Homework Problems

1. Find the slope of the line passing through each pair of points or state that the slope is undefined.

   (a) (2, -2) and (-2, 5) (b) $\left(5, \sqrt{3}\right)$ and $\left(\sqrt{3}, 5\right)$ (c) $\left(-\frac{2}{5}, \frac{1}{3}\right)$ and $\left(\frac{3}{5}, -\frac{5}{3}\right)$

   (d) (-8, 5) and (-8, -3) (e) (0, 0) and (-3, 5) (f) (5, *b*) and (-3, *b*)

   (g) (0, *b*) and (*a, a* + *b*) where $a \neq 0$ (h) (*a* + *b, c*) and (*b* + *c, a*) where $c \neq a$

2. Refer to the federal minimum wage rates in the table below to determine the average rate of change in the minimum wage for the given time periods. (Round to nearest cent.)

Year	**1955**	**1963**	**1967**	**1975**	**1989**	**1997**	**2009**	**2012**
Minimum Wage	$.75	1.25	$1.40	$2.10	$3.35	$5.15	$7.25	$7.25

   (a) 1955 to 2012 (b) 1955 to 1975 (c) 1975 to 2012 (d) 1989 to 1997 (e) 2009 to 2012

3. Find the average rate of change on the graph of $y = (x+4)^2$ from

   (a) $x_1 = -5$ to $x_2 = -4$ (b) $x_1 = -6$ to $x_2 = -4$ (c) $x_1 = 6$ to $x_2 = 8$ (d) $x_1 = -8$ to $x_2 = 0$

4. Find the average rate of change on the graph of $y = 2x^2 - 3x + 5$ from $x_1 = -4$ to $x_2 = -2$.

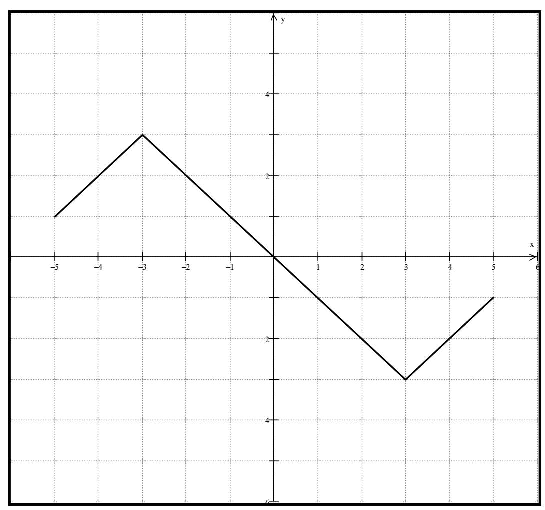
5. Refer to the graph to find the average rate of change from

   (a) $x_1 = -5$ to $x_2 = 5$

   (b) $x_1 = -5$ to $x_2 = -1$

   (c) $x_1 = -3$ to $x_2 = 3$

   (d) $x_1 = 3$ to $x_2 = 5$



2.2 Homework Answers: 1. (a) $-\frac{7}{4}$; (b) -1; (c) -2; (d) undefined; (e) $-\frac{5}{3}$; (f) 0; (g) 1; (h) -1
2. (a) $.11/year; (b) $.07/year; (c) $.14/year (d) $.22 or $.23/year; (e) $0.00/year **3.** (a) -1; (b) -2; (c) 22; (d) 0 **4.** -15 **5.** (a) $-\frac{1}{5}$; (b) 0; (c) -1; (d) 1

2.3 Writing Equations of Lines

In this section you will learn to

- use point-slope form to write an equation of a line
- use slope-intercept form to write an equation of a line
- graph linear equations using the slope and *y*-intercept
- find the slopes and equations of parallel and perpendicular lines
- recognize and use the standard form of a line

Point-Slope Form	Slope-Intercept Form
The point-slope form of the equation of a nonvertical line with slope *m* that passes through (x_1, y_1) is $$y - y_1 = m(x - x_1)$$	The slope-intercept form of the equation of a nonvertical line with slope *m* and *y*-intercept *b* is $$y = mx + b$$

Example 1: Find an equation for the line that passes through the point (-2, 7) and has a slope equal to -5. Write the equation in **point-slope form**. Then write the equation in **slope-intercept form**. (Solve for *y*.)

<u>**Steps**</u>:

1. Substitute x_1, y_1, and *m* values.
2. Simplify and solve for *y*.
3. Check given point and slope (*m*).

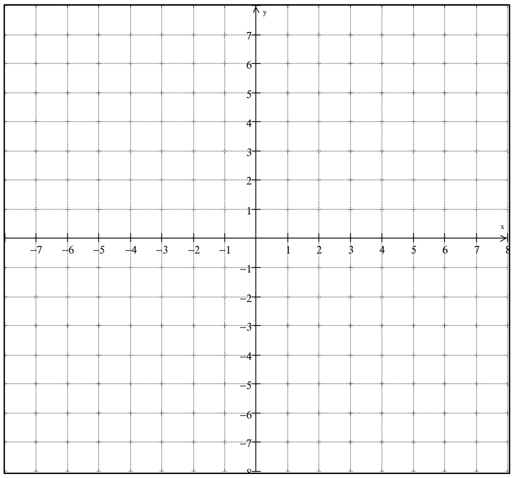
Example 2: Find an equation for the line passing through the points (6, -8) and (4, -3). Write your equation in **point-slope form** and then in **slope-intercept** form.

<u>**Steps:**</u>

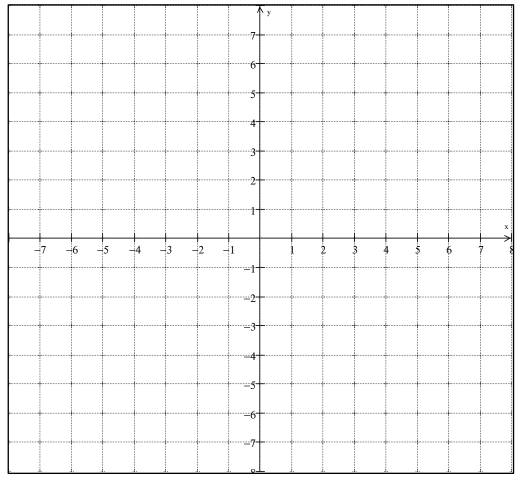
1. Find the slope.
2. Substitute the slope and the values for one of the points.
3. Simplify and solve for *y*.
4. Check the point and slope.

Example 3: Graph each of the following equations.

(a) $y = 3x + 2$ and $y = \frac{1}{3}x + 2$



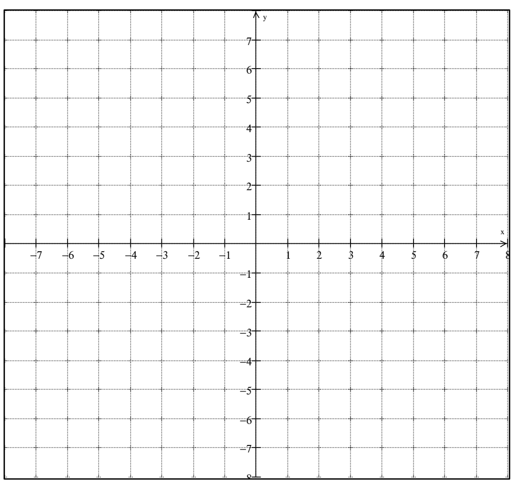
(b) $y = -\frac{1}{3}x - 2$ and $y = -3x - 2$



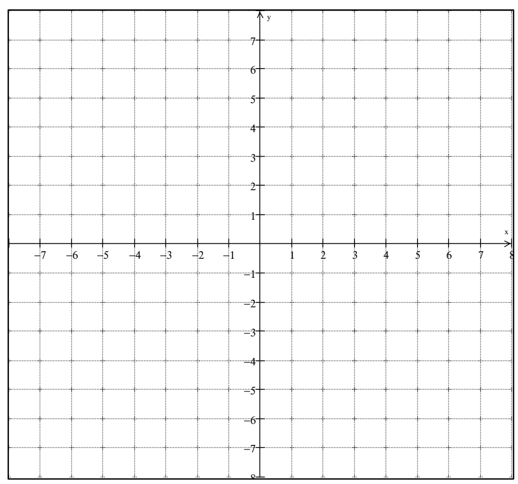
S**teps**:

1. Plot the y-intercept.
2. Use the slope $\left(m = \frac{rise}{run} \right)$ to find a 2^{nd} point.
3. Draw a line through the points.

(c) $f(x) = \frac{2}{5}x - 1$



(d) $x = 4$ and $y = 6$



Horizontal Line Equations:

Vertical Line Equations:

Example 4 (Optional)**:** Find an equation for the line passing through (-3, 4) and m = 2 using the

Point-Slope Method	Slope-Intercept Method

General Form of the Equation of a Line: Every line has an equation that can be written in the general for $Ax + By + C = 0$, where *A, B*, and *C* are real numbers, and *A* and *B* are not both zero.

(Note: Solve the equation for *y* to find the slope and *y*-intercept.)

Example 5: Find the slope and the *y*-intercept for the line whose equation is $3x + 5y + 10 = 0$.

Example 6: Find the slope and the *y*-intercept for the line whose equation is $Ax + By + C = 0$

Parallel Lines	Perpendicular Lines
Slopes are equal.* $m_1 = m_2$	Slopes are negative reciprocals.* $m_1 = -\frac{1}{m_2}$ The product of their slopes is -1.*
Vertical lines (undefined slopes) are parallel.	A horizontal line with slope = 0 is perpendicular to a vertical line with an undefined slope.

*If the lines are not vertical lines.

Example 7: Find an equation for the line through the point (-1, 3) and parallel to the line whose equation is $3x - 2y - 5 = 0$. Write the equation in slope-intercept form.

Example 8: Determine whether the graphs of the equations below are parallel, perpendicular or neither.

(a) $4x+8y-10=0$ and $4y=12-2x$

(b) $3x+2y-7=0$ and $y=\frac{2}{3}x+3$

Example 9: Complete the table below for the **perpendicular lines** l_1 and l_2.

Slope of l_1		$-\frac{2}{3}$	$\frac{1}{3}$		undefined		$2\frac{1}{3}$
Slope of l_2	5			0		.25	

Example 10: Find an equation for the line passing through (1, -2) and perpendicular to $3x - 2y - 4 = 0$.

Example 11: Find an equation for the line passing through the point (-5, 6) and perpendicular to the graph of the line $3x - 24 = 0$.

2.3 Homework Problems:

1. Find an equation for each line based on the conditions below. Write the equation in slope-intercept form and also standard form.

   (a) passing through (-3, 2); $m = -4$

   (b) passing through (-4, 7); $m = -\frac{3}{2}$

   (c) x-intercept (1, 0); $m = -\frac{1}{2}$

   (d) passing through (-1, 1); $m = \frac{2}{5}$

   (e) passing through (-7, 5); $m = 0$

   (f) passing through (8, -1); slope is undefined

2. Write an equation for the line that passes through the two points. Write the answer in slope-intercept form.

   (a) (-5, 6) and (6, -5)

   (b) (4, 0) and (6, -8)

   (c) (3, -4) and (11, -1)

   (d) $\left(-\frac{2}{3}, 4\right)$ and $\left(-\frac{2}{3}, -7\right)$

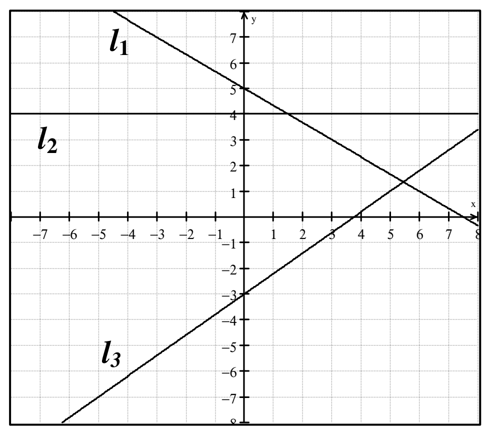
   (e) (2, 3.2) and (5, 5)

   (f) (5, a) and (-7, a)

3. Find an equation for the line that has the following intercepts. Write the equation in standard form.

   (a) (0, 2) and (4, 0)

   (b) $(0, -5)$ and $\left(\frac{15}{2}, 0\right)$



4. Find the equations for the lines $l_1 - l_3$ on the graph at the right.

5. Find the slope and y-intercept for each of the lines below.

   (a) $3x + 7y = 21$

   (b) $8y + 2x = 5$

   (c) $2x + 6 = 2(y + x)$

   (d) $Bx = Cy + A$

6. Determine whether the lines are parallel, perpendicular, or neither.

   (a) $y = 3x + 2$ and $6x - 2y = 5$

   (b) $4x + 8y = 10$ and $4x - 2y = 21$

   (c) $3x + 7y = 21$ and $3x - 7y = 21$

   (d) $\frac{x}{5} + \frac{y}{2} = 1$ and $5x - 2y = 3$

7. Use the given conditions to write an equation for each line in slope-intercept form.

   (a) passing through (-8, -10) and parallel to the line whose equation is $y + 3 = -4x$

   (b) passing through (2, -3) and perpendicular to the line whose equation is $\frac{1}{5}x - y + 6 = 0$

(c) passing through (-2, 2) and parallel to the line whose equation is $3x-2y=5$

(d) passing through (4, -7) and perpendicular to the line whose equation is $x-3=2y$

(e) passing through (2, -3) and perpendicular to $x=4$

(f) passing through (-6, 4) and is perpendicular to the line with an *x*-intercept of 2 and a *y*-intercept of -4

(g) perpendicular to the line whose equation is $3x-2y-4=0$ and has the same *y*-intercept as this line

8. Find the *x* and *y* values if the line through the given points has the indicated slope.

(a) $(x, 7)$, $(-1, y)$, and $(1, 4)$; $m=\frac{3}{2}$

(b) $(x, 9)$, $(-4, y)$, and $(-5, 3)$; $m=-3$

9. Find the coefficients *a* and *b* for the equation $ax+by=30$ so that the graph of the line will have an *x*-intercept of 5 and a *y*-intercept of -3. (Use the definition of intercepts to find *a* and *b*.)

10. The minimum wage at ABC Department Store in 1965 was $1.12. The minimum wage for this store in 2007 was $7.42. (Note: Round all values for this problem to nearest hundredths.)

(a) Use this information to find the equation of the line that models this data in point-slope form.

(b) Use this information to find the equation of the line that models this data in slope-intercept form.

(c) Use your model to predict the minimum wage for 2012.

(d) What is the average rate of change in minimum wage from 1965 to 2007?

2.3 Homework Answers: 1. (a) $y=-4x-10$; $4x+y+10=0$; (b) $y=-\frac{3}{2}x+1$; $3x+2y-2=0$; (c) $y=-\frac{1}{2}x+\frac{1}{2}$; $x+2y-1=0$; (d) $y=\frac{2}{5}x+\frac{7}{5}$; $2x-5y+7=0$; (e) $y=5$; $y-5=0$; (f) $x-8=0$

2. (a) $y=-x+1$; (b) $y=-4x+16$; (c) $y=\frac{3}{8}x-\frac{41}{8}$; (d) $x=-\frac{2}{3}$; (e) $y=\frac{3}{5}x+2$; (f) $y=a$

3. (a) $x+2y-4=0$; (b) $2x-3y=15$ **4.** $l_1: y=-\frac{2}{3}x+5$; $l_2: y=4$; $l_3=\frac{4}{5}x-3$ **5.** (a) $-\frac{3}{7}; 3$ (b) $-\frac{1}{4}; \frac{5}{8}$ (c) 0; 3 (d) $m=\frac{B}{C}; -\frac{A}{C}$ **6.** (a) parallel; (b) perpendicular; (c) neither; (d) perpendicular

7. (a) $y=-4x-42$; (b) $y=-5x+7$; (c) $y=\frac{3}{2}x+5$; (d) $y=-2x+1$; (e) $y=-3$; (f) $y=-\frac{1}{2}x+1$; (g) $y=-\frac{2}{3}x-2$ **8.** (a) $x=3$; $y=1$; (b) $x=-7$; $y=0$ **9.** $a=6$; $b=-10$

10. (a) $(y-1.12)=.15(x-1965)$ or $(y-7.42)=.15(x-2007)$; (b) $y=.15x-293.63$; (c) $y=\$8.17$;

(d) $.15/year

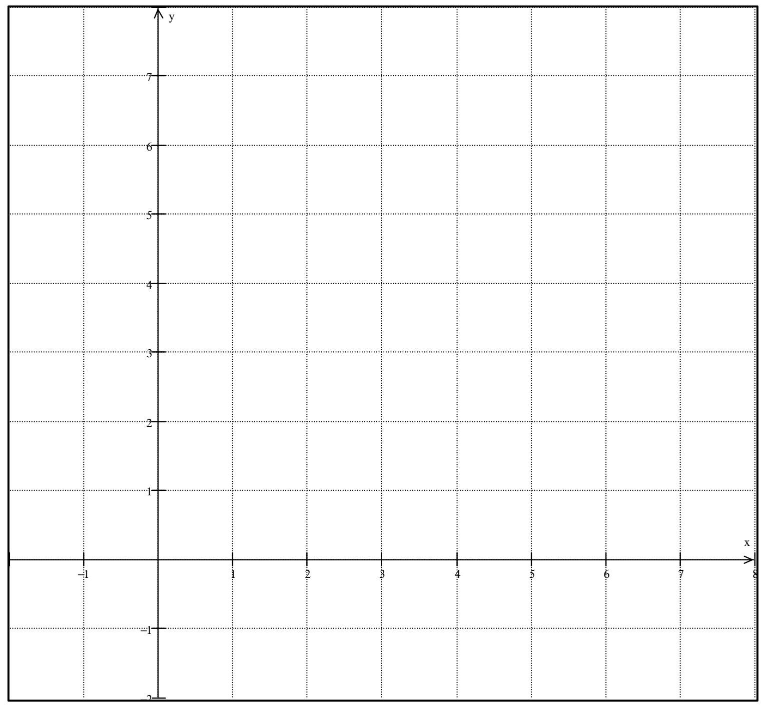
2.4 Distance and Midpoint Formulas; Circles

In this section you will learn to:

- find the distance between two points
- find the midpoint of a line segment
- find the center and radius of a circle
- convert the general form of a circle's equation to standard form

Distance Formula: The **distance**, d, between the points (x_1, y_1) and (x_2, y_2) in the rectangular coordinate system is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Recall: Pythagorean Theorem for right triangles:

$$a^2 + b^2 = c^2$$

Example 1: Use the Pythagorean Theorem to find the distance from (2, 3) to (5, 7).

Example 2: Find the distance between the following sets of points using the distance formula.

(a) (-4, -1) and (2, -3)

(b) $\left(-\frac{1}{4}, -\frac{1}{7}\right)$ and $\left(\frac{3}{4}, \frac{6}{7}\right)$

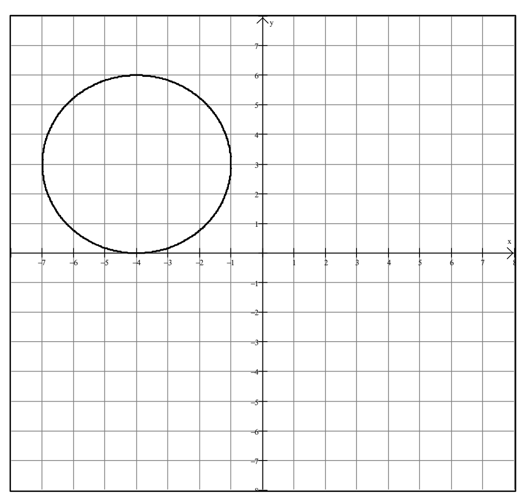
Midpoint Formula: The coordinates of the **midpoint** of a line segment with endpoints (x_1, y_1) and (x_2, y_2) are: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ Note: When finding the **midpoint** of a line segment, you are finding the "**average**" of the *x*- and *y*-values.

Example 3: Find the midpoint of a line segment whose endpoints are $\left(-5\sqrt{3}, -\frac{1}{3}\right)$ and $\left(\sqrt{3}, \frac{3}{5}\right)$.

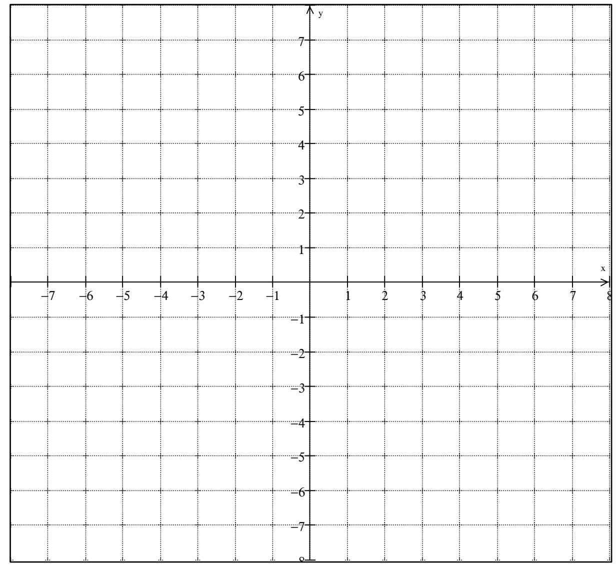
Definition of a Circle: A **circle** is the set of all points in a plane that are equidistant (same distance) from a fixed point, called the **center.** This fixed distance from the center of the circle is called the **radius.**	
Standard Form of the Equation of a Circle: $(x-h)^2 + (y-k)^2 = r^2$ (with center (*h, k*) and radius = *r*)	**General From of the Equation of a Circle:** $x^2 + y^2 + Dx + Ey + F = 0$ (*D, E,* and *F* are real numbers)

Example 4: Find the equation of a circle with center (-2, 3) and radius = $3\sqrt{2}$. Write the equation in standard and general form.

Example 5: Find the equation of the circle shown at the right. Write the equation in standard and general form. Write its domain (*x*-values) and range (*y*-values) using interval notation.



Example 6: The endpoints of the diameter of a circle are (-1, 4) and (5, -4). Find the equation of the circle and write its domain (*x*-values) and range (*y*-values) using interval notation.



Recall **Perfect Square Trinomials:** $x^2 - 6x + 9 = (x-3)^2$ $\quad$ $x^2 + 10x + 25 = (x+5)^2$

$x^2 - 8x + \_\_\_\_ = (x-4)^2$ $\quad$ $x^2 + 14x + \_\_\_\_ = (x+7)^2$

Example 7: Write the equation $x^2 + y^2 - 6x + 10y + 18 = 0$ in standard form. Then find the center and radius of the circle. Also find the domain and range of the circle.

<u>Steps</u>:

1. Move constant term to right side; group *x* and *y* terms on left side.
2. "Form" perfect square trinomials (completing the square).
3. Factor on left side; add on right side.
4. Find center (*h, k*) and radius.

2.4 Homework Problems

1. Find the distance between the points.

   (a) (4, 9) and (9, 21) (b) (-1, 4) and (3, -2)

   (c) $\left(3\sqrt{3}, \sqrt{5}\right)$ and $\left(-\sqrt{3}, 4\sqrt{5}\right)$ (d) $\left(\frac{7}{3}, \frac{1}{5}\right)$ and $\left(\frac{1}{3}, \frac{6}{5}\right)$

2. Find the midpoint of the line segment with the endpoints given below.

   (a) (1, -6) and (-8, -4) (b) (-3, -4) and (6, -8)

   (c) $\left(7\sqrt{3}, -6\right)$ and $\left(\sqrt{3}, -2\right)$ (d) $\left(\frac{7}{3}, \frac{1}{5}\right)$ and $\left(\frac{1}{3}, \frac{6}{5}\right)$

3. Write the standard and the general form for the equation of the circle with the given center and radius.

   (a) (0, 0); $r = \sqrt{7}$ (b) (-3, 1); $r = 2$ (c) (-4, 0); $r = 5\sqrt{3}$

4. The endpoints of the diameter of a circle are (-3, 2) and (3, 10).

   (a) Find the coordinates of the circle's center.
   (b) Find the radius of the circle.
   (c) Write the equation of the circle in standard form.
   (d) Write the equation of the circle in general form.
   (e) Find the domain (x-values) and range (y-values) of the circle using interval notation.

5. Find the center, radius, domain, and range of each circle.

   (a) $x^2 + y^2 = 64$ (b) $(x+6)^2 + y^2 = \frac{1}{9}$ (c) $(x+11)^2 + (x-7)^2 = 121$

6. Find the center and radius of each circle.

   (a) $x^2 + y^2 - 4x + 8y + 11 = 0$ (b) $x^2 + y^2 + 4x - 6y - 23 = 0$

   (c) $x^2 + y^2 - x + 2y + 1 = 0$ (d) $x^2 + y^2 - 3x - 2y - 1 = 0$

2.4 Homework Answers: 1. (a) 13; (b) $2\sqrt{13}$; (c) $\sqrt{93}$; (d) $\sqrt{5}$ **2.** (a) $\left(-\frac{7}{2}, -5\right)$; (b) $\left(\frac{3}{2}, -6\right)$; (c) $(4\sqrt{3}, -4)$; (d) $\left(\frac{4}{3}, \frac{7}{10}\right)$ **3.** (a) $x^2 + y^2 = 7$; $x^2 + y^2 - 7 = 0$; (b) $(x+3)^2 + (y-1)^2 = 4$; $x^2 + y^2 + 6x - 2y + 6 = 0$; (c) $(x+4)^2 + y^2 = 75$; $x^2 + y^2 + 8x - 59 = 0$ **4.** (a) (0, 6); (b) 5; (c) $x^2 + (y-6)^2 = 25$; $x^2 + y^2 - 12y + 11 = 0$; (e) D: [-5, 5]; R: [1, 11] **5.** (a) (0, 0); r = 8; D: [-8, 8]; R: [-8, 8]; (b) (-6, 0); $r = \frac{1}{3}$; D: $\left[-\frac{19}{3}, -\frac{17}{3}\right]$; R: $\left[-\frac{1}{3}, \frac{1}{3}\right]$; (c) (-11, 7); r = 11; D: [-22, 0]; R: [-4, 18]

6. (a) (2, -4); 3 (b) (-2, 3); 6 (c) $\left(\frac{1}{2}, 1\right)$; $\frac{1}{2}$ (d) $\left(\frac{3}{2}, 1\right)$; $\frac{\sqrt{17}}{2}$