The final exam will cover chapters 16, 17, and 18. However, this practice final only has questions relating to material covered after the second midterm (17.4, 17.5 and chapter 18). For additional practice with earlier material, please consult the midterms, practice midterms, and textbook questions.

Exam Tips:

- Know the hypothesis and conclusions for all theorems covered so far as well as the tricky ways we can apply the main theorems (Greene's, Stokes', and divergence).
- Know all relevant definitions.
- In addition to the problems provided here, it would be a good idea to do some additional exercises from the textbook (use the homework as an indication of which problem types are relevant),

Practice Problems:

1. Write down Greene's theorem including all hypothesis and its conclusion. Write down the definitions of any terms which appear in the theorem.
2. Repeat problem 1 with Stokes' theorem.
3. Repeat problem 1 with the divergence theorem.
4. Let $\mathcal{S}$ be the portion of the sphere $x^2 + y^2 + z^2 = 9$ where $1 \leq x^2 + y^2 \leq 4$ and $z \geq 0$. Compute
$$\iint_{\mathcal{S}} \frac{1}{z} dS.$$
5. Consider the the hemisphere $\mathcal{S}$ of radius 1 with parametrization
$$G(\phi, \theta) = \langle \cos\theta \sin\phi,\ \sin\theta \sin\phi,\ \cos\phi \rangle \qquad 0 \leq \theta \leq 2\pi,\ 0 \leq \phi \leq \frac{\pi}{2}.$$
Its boundary $\partial\mathcal{S}$ is the circle $x^2 + y^2 = 1$ in the xy-plane. Determine if the following paramaterization of the $\partial\mathcal{S}$ is compatible with the given parametrization of $\mathcal{S}$ (i.e. will using these two parametrizations to compute both sides of Stokes' theorem give us a sign error or not?):
$$\vec{c}(t)\ \langle \cos t,\ \sin t,\ 0 \rangle \qquad 0 \leq t \leq 2\pi.$$
6. For the cone $\mathcal{S}$ given by $z^2 = x^2 + y^2$, $z \leq 4$. Write a parametrization for $\mathcal{S}$ with upward pointing normal vector. Write a parametrization for $\partial\mathcal{S}$ which is compatible with the orientation of $\mathcal{S}$.

7. Let $\mathcal{S}$ be the boundary of the unit cube $[0,1]\times[0,1]\times[0,1]$ (that is, $\mathcal{S}$ is the union of the six faces of the cube). Compute

$$\iint_{\mathcal{S}} \left\langle 8x^3y, 5z, 12zy^2 \right\rangle \cdot d\vec{S}.$$

8. Let $\mathcal{D}$ be a region in $\mathbb{R}^2$, and suppose $\partial\mathcal{D}$ consists of two curves: an outer curve $\mathcal{C}_1$ and an inner curve $\mathcal{C}_2$ (around a hole in $\mathcal{D}$), both with the boundary orientation. Let $\vec{F} = \langle e^{x+y},\ e^{x+y}\rangle$ and suppose we know that $\int_{\mathcal{C}_2} \vec{F}\cdot d\vec{s} = 4$. Use Greene's theorem to compute $\int_{\mathcal{C}_1} \vec{F}\cdot d\vec{s}$.

9. Let $\mathcal{C}$ be the close curve consisting of line segments joining points $(1,0,0)$, $(0,2,0)$, and $(0,0,3)$ (in that order. Compute

$$\int_{\mathcal{C}} \langle 2yz,\ 0,\ xy\rangle \cdot d\vec{s}.$$

10. Let

$$\vec{F}(x,y,z) = \left\langle \frac{x}{(x^2+y^2+z^2)^{3/2}},\ \frac{y}{(x^2+y^2+z^2)^{3/2}},\ \frac{z}{(x^2+y^2+z^2)^{3/2}} \right\rangle,$$

be the inverse-square field. Let $\mathcal{S}$ be a closed surface and show

$$\iint_{\mathcal{S}} \vec{F}\cdot d\vec{S} = \begin{cases} 4\pi & \text{if } \mathcal{S} \text{ encloses } (0,0,0) \\ 0 & \text{otherwise} \end{cases}$$

by first computing the flux of $\vec{F}$ through a sphere of radius R and then using the divergence theorem.