Midterm 1

Name: ____________________

NetID: ____________________

Instructions:

1. You will have 50 minutes to complete the exam.
2. The exam is a total of 3 questions, their respective points values are listed below.
3. Unless stated otherwise, you must justify your answers with proofs.
4. You may cite any results from lecture or the homework.
5. No books, notes, calculators, or electronic devices are permitted.
6. If you require additional space, please use the reverse side of the pages.
7. The exam has a total of 4 pages, please verify that your copy has all 4 pages.

Question	Score	Points
1.		10
2.		25
3.		15
Total		50

1. Let X be a nonempty set and $\mathcal{E}$ a collection of subsets. Show that

$$\mathcal{M}(\mathcal{E}) = \bigcup_{\mathcal{F}} \mathcal{M}(\mathcal{F})$$

where the union is over countable subcollections $\mathcal{F} \subset \mathcal{E}$.

2. For a measure space $(X, \mathcal{M}, \mu)$, a set $A \in \mathcal{M}$ is called an **atom** for μ if $\mu(A) > 0$ and for any $B \in \mathcal{M}$ with $B \subset A$ one has either $\mu(B) = 0$ or $\mu(B) = \mu(A)$. Let μ_F be the Lebesgue–Stieltjes measure associated to an increasing, right-continuous function $F \colon \mathbb{R} \to \mathbb{R}$.

   (a) Show that for $x \in \mathbb{R}$, $A = \{x\}$ is an atom for μ_F if and only if F is **not** left-continuous at x.

   (b) Show that if F is continuous, then μ_F has no atoms. [**Hint:** use dyadic h-intervals $(\frac{k}{2^n}, \frac{k+1}{2^n}]$.]

3. Let $(X, \mathcal{M})$ be a measurable space and let $f, g\colon X \to \overline{\mathbb{R}}$ be $\mathcal{M}$-measurable. For $a, b \in \mathbb{R}$ with $a < b$, show that the set
$$\{x \in X \colon f(x) + a \leq g(x) \leq f(x) + b\}$$
is in $\mathcal{M}$.