Midterm 1

Name: ______________________________

NetID: ______________________________

Instructions:

1. You will have 50 minutes to complete the exam.
2. The exam is a total of 3 questions, their respective points values are listed below.
3. Unless stated otherwise, you must justify your answers with proofs.
4. You may cite any results from lecture or the homework.
5. No books, notes, calculators, or electronic devices are permitted.
6. If you require additional space, please use the reverse side of the pages.
7. The exam has a total of 4 pages, please verify that your copy has all 4 pages.

Question	**Score**	**Points**
1.		25
2.		15
3.		10
Total		50

1. Let $(X, \mathcal{M})$ be a measurable space.

   (a) For each $n \in \mathbb{N}$, let μ_n be a measure on $\mathcal{M}$ and let $\alpha_n \in [0, +\infty)$. Show that

   $$\mu(E) := \sum_{n=1}^{\infty} \alpha_n \mu_n(E)$$

   defines a measure on $\mathcal{M}$.

   (b) Let μ be a measure on $\mathcal{M}$ and $E_0 \in \mathcal{M}$. Show that $\nu(E) := \mu(E \cap E_0)$ defines a measure on $\mathcal{M}$.

   (c) Given a σ-finite measure μ on $\mathcal{M}$, show that there exists a finite measure ν on $\mathcal{M}$ satisfying that $E \in \mathcal{M}$ is μ-null if and only if it is ν-null. (We say μ and ν are **equivalent** in this case.)

2. Suppose $F\colon \mathbb{R} \to \mathbb{R}$ is an increasing, differentiable function with $\sup_{t\in\mathbb{R}} F'(t) < \infty$, and let μ_F be the associated Lebesgue–Stieltjes measure. Denote the Lebesgue measure by m.
   (a) Show that every m-null set is μ_F-null.
   (b) Find an example of such a function F for which the converse is false.

3. Suppose $f\colon \mathbb{R} \to \mathbb{R}$ has a countable discontinuity set. Show that f is Borel measurable.
   [**Hint:** the ϵ-δ definition of continuity will be more helpful than the sequential definition.]