Name:____

Student ID Number:______________________________

Instructions:

1. You will have 50 minutes to complete the exam.
2. The exam is a total of 4 questions and each question is worth 10 points.
3. There are two survey questions on Page 6 worth 1 bonus point each.
4. Unless stated otherwise, you may use results we proved in class and on the homework.
5. No books, notes, calculators, or electronic devices are permitted.
6. If you require additional space, please use the reverse side of the pages.
7. The exam has a total of 7 pages with the last page left blank for scratch work. Please verify that your copy has all 7 pages.

Question	**Score**	**Points**
1.		10
2.		10
3.		10
4.		10
Survey		+2
Total		40

1. (a) For (E, d) a metric space, $(x_n)_{n\in\mathbb{N}} \subset E$ a sequence, and $x \in E$ a point, state what it means for the sequence $(x_n)_{n\in\mathbb{N}}$ to **converge** to x.

   (b) In the metric space $(\mathbb{R}, d)$, where $d(x, y) = |x - y|$, find the limit of the following sequence and prove it converges: $\left(\dfrac{3n^3}{n^3+n}\right)_{n\in\mathbb{N}}$.

2. (a) For a set E and a map $d\colon E \times E \to \mathbb{R}$, state the **four** conditions required for a (E, d) to be a **metric space**.

   (b) For $x, y \in \mathbb{R}$ define

$$d(x, y) := \min\{|x - y|, 1\}.$$

   Prove that $(\mathbb{R}, d)$ is a metric space.

3. (a) In a metric space (E, d), state what it means for a subset $S \subset E$ to be **open**.

   (b) In the metric space $(\mathbb{R}^2, d_2)$, where d_2 is the 2-dimensional Euclidean metric, prove the subset

$$S := \{(x, y) \in \mathbb{R}^2 \colon -5 < x < 5\}$$

   is open.

4. (a) In a metric space (E, d), state what it means for a subset $S \subset E$ to be **closed**.

   (b) In the metric space $(\mathbb{R}^2, d_\infty)$, where

$$d_\infty((x_1, y_1), (x_2, y_2)) = \max\{|x_1 - x_2|, |y_1 - y_2|\} \qquad (x_1, y_1), (x_2, y_2) \in \mathbb{R}^2,$$

   prove that the subset

$$S := \{(x, y) \in \mathbb{R}^2 \colon y \cdot x \geq 1\}$$

   is closed.

   [Hint: for a convergence sequence $((x_n, y_n))_{n\in\mathbb{N}} \subset S$, consider the sequences $(x_n)_{n\in\mathbb{N}}, (y_n)_{n\in\mathbb{N}} \subset \mathbb{R}$].

The following (optional) questions are worth 1 bonus point each:

- Rate the pacing of lectures (too slow, too fast, just right, etc.)

- Rate the difficulty of homework assignments (too easy, too hard, just right, etc.).

Scratch Work.