C*.1 Recall that the set $U(\mathcal{H})$ of unitaries in $B(\mathcal{H})$ is a group under multiplication. A *unitary representation* of a group G is a group homomorphism $\rho : G \to U(\mathcal{H})$. Show that representations of $\mathbb{C}G$ are in bijection with unitary representations of G.

C*.2 Complete the proof of Proposition 6.3 by showing that ψ is well-defined (independent of the choice of sequence $(a_n)_n$); $*$-preserving; and multiplicative.

W*.1 Suppose $(x_i)_{i\in I} \subset B(\mathcal{H})$ is a uniformly bounded net: $\sup_i \|x_i\| < \infty$.

(a) Show that $(x_i)_{i\in I}$ converges in the σ-SOT if and only if it converges in the SOT.

(b) Show that $(x_i)_{i\in I}$ converges in the σ-WOT if and only if it converges in the WOT.

(c) Show that the example $(x_{m,n})_{m\leq n}$ defined in lecture is **not** uniformly bounded.

W*.2 Recall that a $*$-isomorphism $\pi : M \to N$ between von Neumann algebras $M \subset B(\mathcal{H})$ and $N \subset B(\mathcal{K})$ is called a *spatial isomorphism* if there exists a unitary $U : \mathcal{H} \to \mathcal{K}$ such that $\pi(x) = UxU^*$ for all $x \in M$. Show that a spatial isomorphism $\pi\colon M \to N$ is normal.