A locking pattern

Supporting images for a talk by Hari Bercovici

Free probability and the $N \longrightarrow \infty$ *limit*

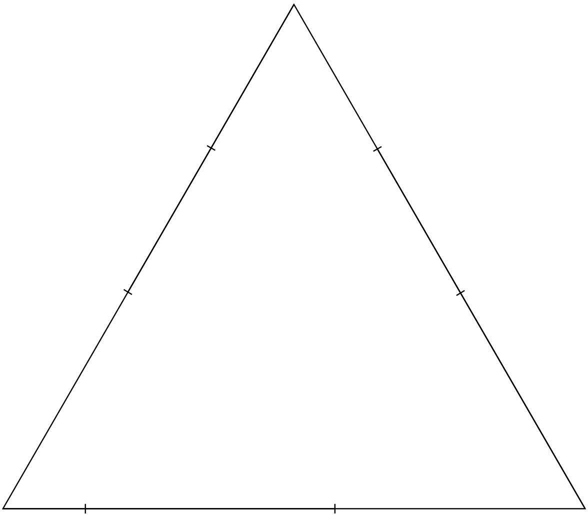
Berkeley, March 1916

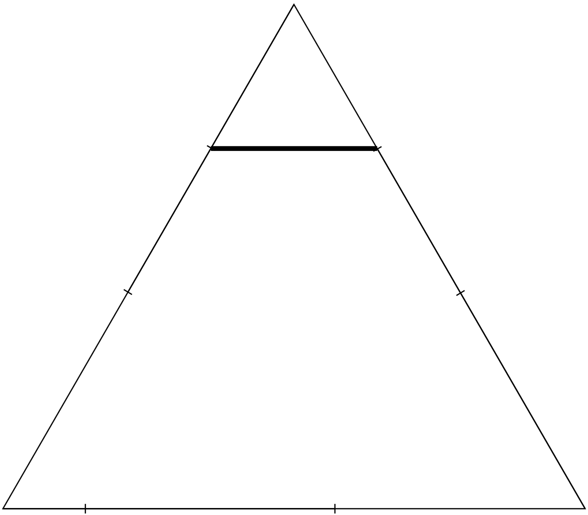
In the following pictures, we must think of three 7×7 matrices A, B, C with eigenvalues $\alpha_1 = \alpha_2 > \alpha_3 = \alpha_4 > \alpha_5 = \alpha_6 = \alpha_7$, $\beta_1 > \beta_2 = \beta_3 = \beta_4 > \beta_5 = \beta_6 = \beta_7$, $\gamma_1 = \gamma_2 > \gamma_3 = \gamma_4 > \gamma_5 = \gamma_6 = \gamma_7$ such that $A + B = C$. A hive associated with these values is a concave function defined on an equilateral triangle of size 7, affine on each unit "lattice" triangle, and with boundary values at lattice points (starting at the top and continuing counterclockwise) equal to

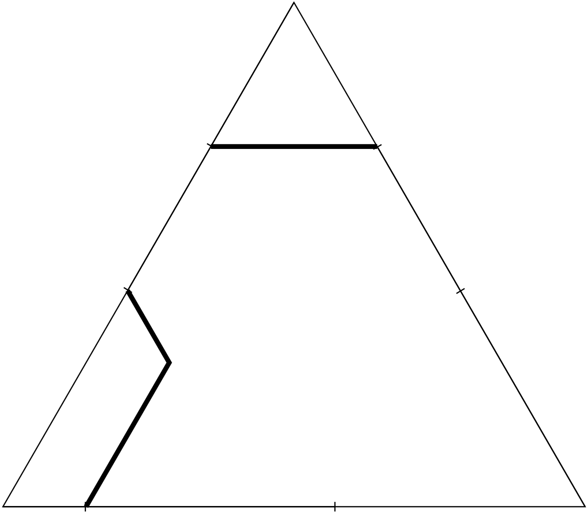
$$\begin{aligned} 0, \alpha_1, \alpha_1 + \alpha_2, \ldots, \alpha_1 + \cdots + \alpha_6, \\ \mathrm{Tr}(A), \mathrm{Tr}(A) + \beta_1, \ldots, \mathrm{Tr}(A) + \beta_1 + \cdots + \beta_6, \\ \mathrm{Tr}(C) = \mathrm{Tr}(A) + \mathrm{Tr}(B), \mathrm{Tr}(C) - \gamma_7, \mathrm{Tr}(C) - \gamma_7 - \gamma_6, \ldots, \gamma_1 \end{aligned}$$

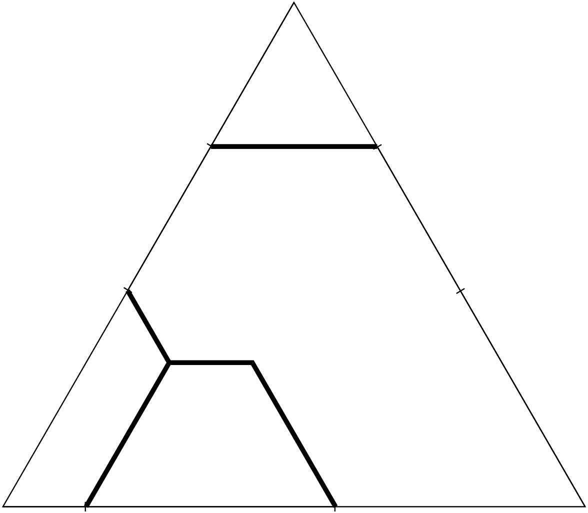
The pictures indicate successively the larger areas on which the hive must be an affine function. Then the red arrows indicate the order in which hive values are determined at the corners of these regions. The differences between the hive values at the end points of the red arrows is then calculated. (The hive is h and the coordinate values should be straightforward.) The last two figures indicate the "canonical" honeycomb associated with this locking patern. The honeycomb shows how to obtain the hive differences more efficiently. These differences are then used in the calculation of certain orthogonal projections whose existence was conjectured by Danilov and Koshevoy.

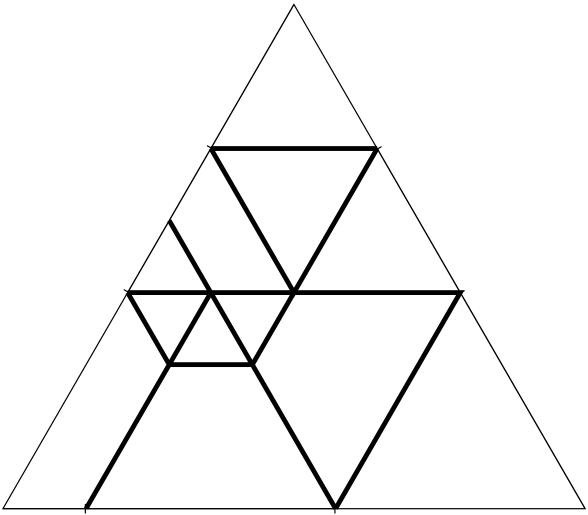
This is part of a joint project with W. S. Li which we hope brings new insight in the Connes embedding problem.

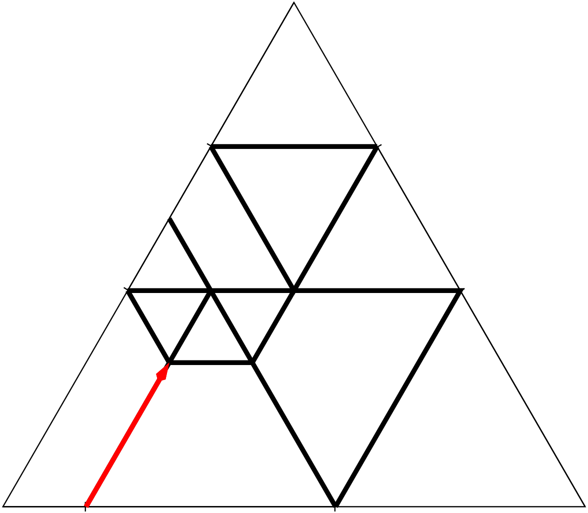


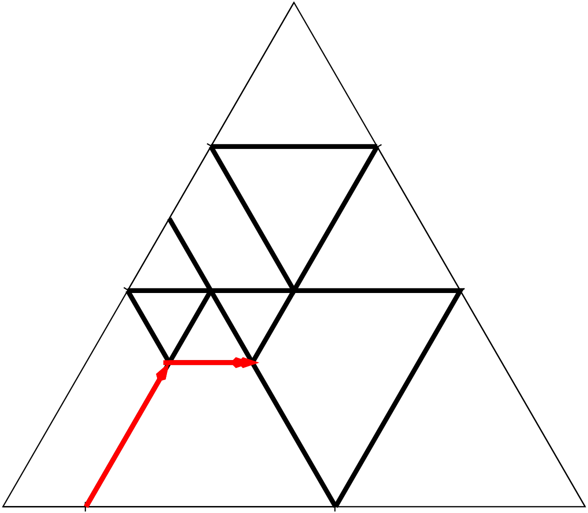


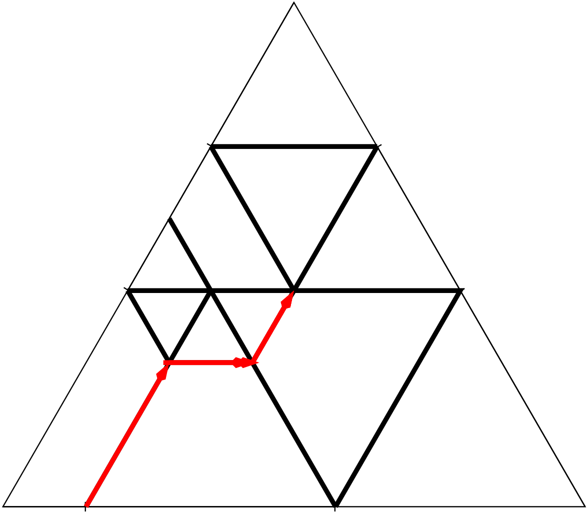


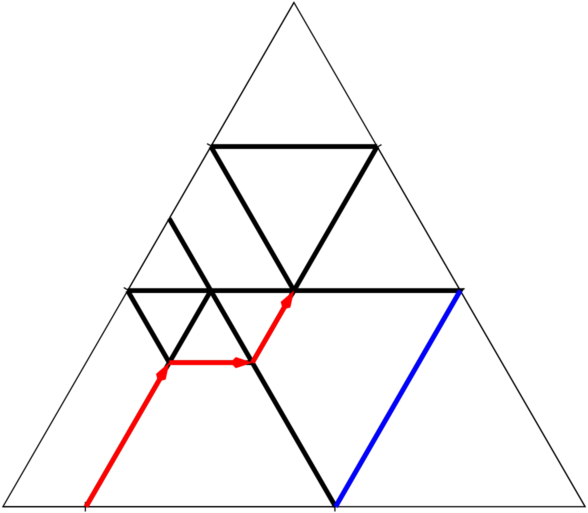


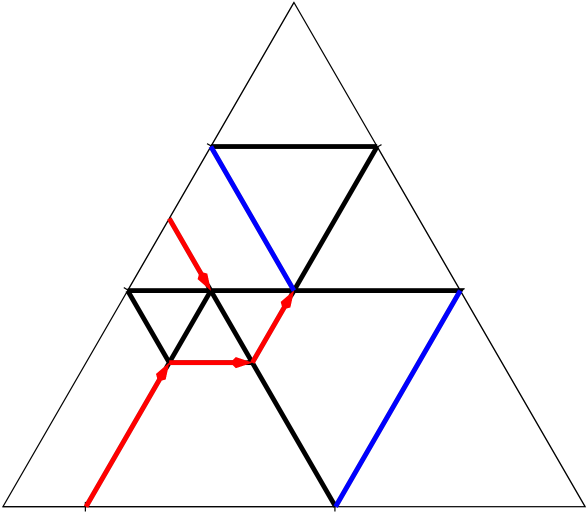


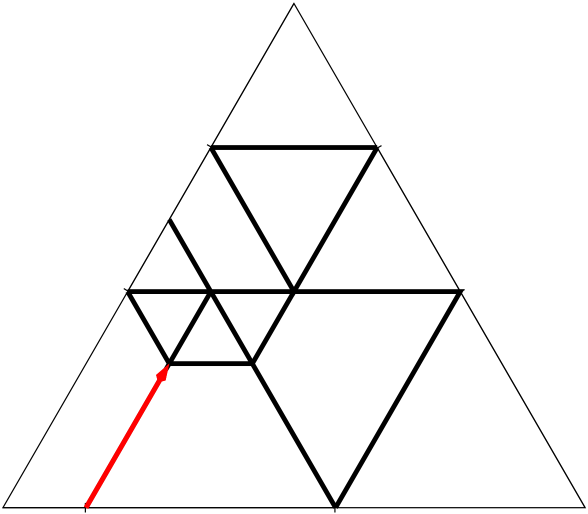




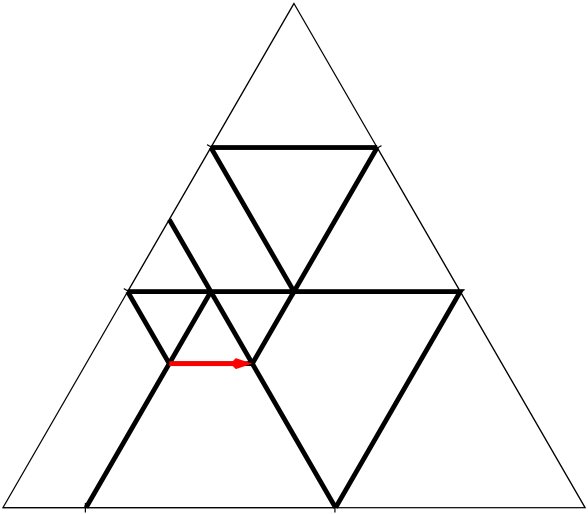




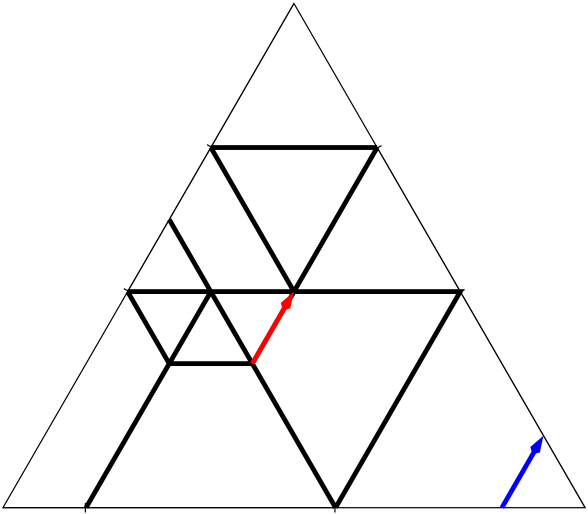




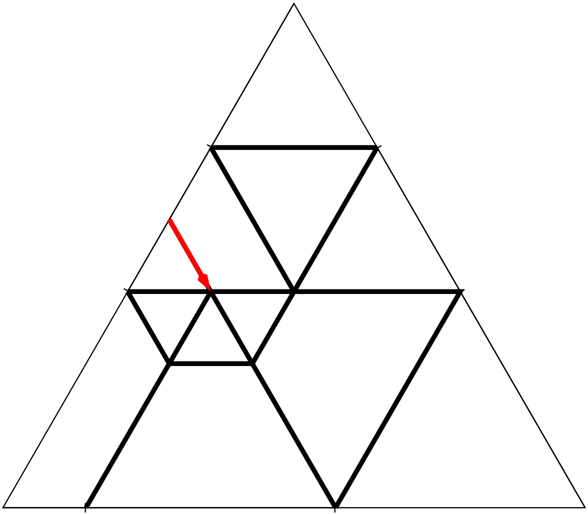
$h(6,1) - h(4,1) = 2\alpha_{5-7}$



$h(3,2) - h(4,1) = \beta_{2-4}$



$h(2,2) - h(3,2) = \beta_{5-7} - \gamma_{5-7}$



$$h(3,0) - h(3,1) = \alpha_{1-2} + \alpha_{5-7} + \beta_{2-4} + \beta_{5-7} - \gamma_{1-2} - \gamma_{3-4} - \gamma_{5-7}$$