# 1.1 Linear and Rational Equations

**In this section you will learn to:**

- recognize equations that are identities, conditional, or contradictions
- solve linear equations in one variable
- find restrictions on variable values
- solve rational equations with variables in the denominator
- solve formulas for a specific value

| Types of Equations | | |
|---|---|---|
| **Identity** | **Conditional** | **Contradiction/Inconsistent** |
| Solution set consists of all real numbers*. | Solution set consists of one or more solutions (but not an identity). | The equation has no solutions. |
| Example: $x+2=5+x-3$<br><br>$2=2$ (or $0=0$)<br>(Both sides are "identical".) | Example: $x^2-5x+6=0$<br><br>$x=2$ or $x=3$ | Example: $x+2=6+x-3$<br><br>$2=3$<br>(Since $2 \neq 3$, there is no solution.) |
| **ALWAYS** TRUE | **SOMETIMES** TRUE | **NEVER** TRUE |

*except when one or both sides of the equation results in division by 0 (Example: $\frac{2x}{x}=2$)

<u>**Solving Linear or First-Degree Equations**</u> ($ax+b=0$, where $a$ and $b$ are real numbers and $a \neq 0$)

**Example 1:** $5x-(2x+2)=x+(3x-5)$

<u>**Steps**</u>**:** (if coefficients are integers)

1. Simplify each side of the equation.
2. Move variable terms to one side and constant terms to the other.
3. Isolate the variable. (Multiply or divide based on the coefficient.)
4. Check the solution in the **original** equation. When "solving" an equation, be sure to write your answer in solution set form.

Solution Set:

Type of Equation: ____________________________

**Example 2:** Solve $3(t+5)+6t=5-3(1-3t)$

Solution Set:

Type of Equation: ______________________________

**Example 3:** Solve: $3+\frac{3x}{2}=5(\frac{x}{6}+1)+\frac{2}{3}x-2$
(Hint: If coefficients are fractions, first multiply sides of the equation by the **Lowest Common Denominator**, or LCD, to clear the fractions.)

Solution Set:

Type of Equation: _________________________

A **rational expression** is the quotient of two polynomials as shown in Example 4 below. The **domain** of a rational expression is the set of real numbers for which the expression is defined. Since division by zero is not defined, numbers that make the denominator zero must be excluded from the domain.

A **rational equation** is an equation that contains rational expressions. When solving this type of equation, you may need to multiply both sides by a quantity (generally the LCD) which may contain a variable. (When the variable appears in the denominator the equation is no longer a linear equation.) However, you must exclude any values of the variable that make the denominator zero as this would give a false solution, called an **extraneous solution.**

**Example 4:** Find all real numbers that must be excluded from the domain of each rational expression.

$\frac{2a}{a-5}$ $\qquad \frac{3}{2z+5}$ $\qquad \frac{3}{m^2-3m-4}$ $\qquad \frac{t-2}{3}$ $\qquad \frac{q-5}{q}$

**Example 5:** Solve $\frac{5}{x} = \frac{10}{3x} + 4$

What value(s) of $x$ must be excluded from the solution? __________

Type of Equation: ________________________________

Solution Set:

Explain the difference between a rational **expression** (Example 4) and a rational **equation** (Example 5).

**Example 6:** Find all values of $x$ for which $y_1 = y_2$ given $y_1 = \frac{22}{x^2 - 16}$ and $y_2 = \frac{1}{x+4} + \frac{1}{x-4}$.

What value(s) of $x$ must be excluded? __________

Type of Equation: __________________________

Solution Set:

**Example 7:** Solve for $n$: $\frac{1}{n-4} - \frac{5}{n+2} = \frac{6}{n^2 - 2n - 8}$

What value(s) of $n$ must be excluded? __________

Type of Equation: __________________________

Solution Set:

**Example 8:** Solve: $\dfrac{b+2}{b+3}=1-\dfrac{1}{b^2+2b-3}$

What value(s) of $b$ must be excluded? __________

Type of Equation: ___________________________

Solution Set:

**Solving Formulas for a Specific Value (or in terms of another variable):**

**Example 9:** Solve $V=\dfrac{1}{3}Bh$ for $B$.

**Steps:**

1. If fractions are involved, multiply by LCD.
2. Move terms with desired variable to one side. Move all other terms to other side.
3. If two or more terms contain the desired variable, FACTOR out the variable.
4. Divide both sides by the "non variable" factor.

**Example 10:** Solve $S = P + Prt$ for $t$.

When solving for $t$, Sam's answer was $t=\dfrac{S-P}{\Pr}$ while Sophia had $t=\dfrac{S}{\Pr}-\dfrac{1}{r}$ for an answer. Which student had the correct answer? Explain.

**Example 11:** Solve $S = P + Prt$ for $P$.

This time Sam's answer was $P = \frac{S}{1+rt}$ and Sophia simplified her answer as $P = S + \frac{S}{rt}$. Which student had the correct answer. Explain.

**Example 12:** Solve $x = \frac{a-b}{c}$ for $b$.

**Example 13:** Solve $C = \frac{A+B}{A-B}$ for $A$.

**Example 14:** A formula for converting temperature from degrees Fahrenheit to degrees Celsius is $C = \frac{5}{9}(F-32)$. Use this formula to convert the temperatures below.

$95°F =$ _______ $°C$

$25°C =$ _______ $°F$

Now solve the formula for $F$ (Fahrenheit in terms of Celsius) and then convert the given temperatures.

$25°C =$ _______ $°F$

$0°C =$ _______ $°F$ (freezing pt. for water)

$100°C =$ _______ $°F$ (boiling pt. for water)

**Example 15:** The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. Solve this formula for *r*.

Now use this formula to estimate the radius of the planet Jupiter whose volume is 1.20 x $10^{16}\,km^3$. Round your answer to the nearest thousand.

**Example 16:** A company wants to increase the 10% peroxide content of its product by adding pure peroxide (100% peroxide). If *x* liters of pure peroxide are added to 500 liters of its 10% solution, the concentration, *C*, of the new mixture is given by

$$C = \frac{x + 0.1(500)}{x + 500}$$

How many liters of pure peroxide should be added to produce a new product that is 28% peroxide? (Consider different ways to solve this problem.)

## 1.1 Homework Problems

1. Find all real numbers that must be excluded from the domain of each rational expression.

(a) $\dfrac{5}{x^2-4}$ (b) $\dfrac{x-3}{x(x+4)}$ (c) $\dfrac{x+3}{3x^3-27x}$ (d) $\dfrac{1}{x^2+3x-4}$

**For Problems 2–8, solve the equation and then determine if it is an example of an identity, contradiction or conditional.**

2. $2x-4(5x+1)=3x+38$

3. $\dfrac{x}{2}-\dfrac{1}{2}-\dfrac{x}{5}=\dfrac{1}{10}$

4. $\dfrac{3x+1}{3}-\dfrac{1-x}{4}=\dfrac{13}{2}$

5. $14x+10=11(x+3)+3x$

6. $\dfrac{8}{x+2}+\dfrac{6}{x}=\dfrac{20}{x^2+2x}$

7. $7x-21=7(x-3)$

8. $\dfrac{x+2}{x+3}=1-\dfrac{1}{x^2+2x-3}$

9. Solve $C=2\pi r$ for $r$.

10. Solve $a=2b-3c$ for $c$.

11. Solve $\dfrac{a}{x}-\dfrac{y}{b}=3$ for $x$.

12. Solve $C=\dfrac{A+B}{A-B}$ for $B$.

13. Solve $A+B=\dfrac{AC+2D}{D}$ for $D$.

14. Solve: $\dfrac{1}{a}-\dfrac{2}{b}=\dfrac{3}{c}$ for $b$.

15. Solve $\dfrac{1}{x}+\dfrac{2}{y}=5$ for $x$.

16. Solve $x=y+\dfrac{2w}{z}$ for $z$.

17. An example of a formula used to model the proportion of correct responses in terms of the number of trials of a particular task is $P=\dfrac{0.9x-0.4}{0.9x+0.1}$, where $P$ is the proportion of correct responses after $x$ trials. How many learning trials are necessary for 0.95 of the responses to be correct?

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**1.1 Homework Answers: 1.**(a) $x\neq\pm 2$ (b) $x\neq -4, 0$ (c) $x\neq 0, \pm 3$ (d) $x\neq -4, 1$ **2**. {-2}; conditional **3**. {2}; conditional **4**. $\left\{\dfrac{77}{15}\right\}$; conditional **5**. $\phi$; contradiction **6**. $\left\{\dfrac{4}{7}\right\}$; conditional **7**. all real numbers; identity **8**. {2}; conditional **9**. $r=\dfrac{C}{2\pi}$ **10**. $c=\dfrac{2b-a}{3}$ **11**. $x=\dfrac{ab}{3b+y}$ **12**. $B=\dfrac{AC-A}{C+1}$ **13**. $D=\dfrac{AC}{A+B-2}$ **14**. $b=\dfrac{2ac}{c-3a}$ **15**. $x=\dfrac{y}{5y-2}$ **16.** $z=\dfrac{2w}{x-y}$ **17.** About 11 learning trials

# 1.2 Applications of Linear Equations

**In this section you will learn to:**

- use linear equations to solve word problems

**Steps/Tips for Solving Word Problems:**

1. **Read** the problem carefully. **Underline** key words and phrases. Let $x$ (or any variable) represent one of the unknown quantities. **Draw** a picture or diagram if possible.

2. If necessary write expressions for **other unknowns** in terms of $x$.

3. Write a **verbal model** of the problem and then replace words with numbers, variables and/or symbols.

4. **Solve** the equation. **Answer** the question in the problem. **Label** answers!

5. **Check** your answer(s) in the original word problem (not in your equation).

**Example 1:** The number of cats in the U. S. exceeds the number of dogs by 7.5 million. The number of cats and dogs combined is 114.7 million. Let $c$ denote the number of cats. Write an equation in terms of the variable $c$ which models this information. Then use your equation to find the number of cats and dogs.

**Example 2:** After a 30% discount, a cell phone sells for \$224. Find the original price of the cell phone before the discount was applied to the purchase.

**Example 3:** Including a 6% sales tax, an item costs \$91.69. Find the cost of the item before the sales tax was added.

**Example 4:** You are choosing between two car rental agencies. Avis charge \$40/day plus \$.10/mile to rent a car. Hertz charges \$50/day plus \$.08/mile. You plan to rent the car for three days. After how many miles of driving will the total cost for each agency be the same?

**Example 5:** What temperature is the same number of degrees in both Celsius and Fahrenheit?

Recall: $C = \frac{5}{9}(F - 32)$ and $F = \frac{9}{5}C + 32$

**Example 6:** The perimeter of a triangular lawn is 162 meters. The length of the first side of a triangle is twice the length of the second side. The length of the third side is 6 meters shorter than three times the length of the second side. Find the lengths of the three sides.

**Example 7:** (Simple Interest Problem) Tricia received an inheritance of \$5500. She invested part of it at 8% simple interest and the remainder at 12% simple interest. At the end of the year she had earned \$540. How much did Tricia invest at each interest amount?

**Example 8:** How many liters of a 9% solution of salt should be added to a 16% solution in order to obtain 350 liters of a 12% solution?

**Example 9:** A student scores 82%, 86% and 78% on her first three exams. What score is needed on the fourth exam for the student to have an average of 85% for all four exams?

**Example 10:** A student scores 85%, 72%, 96%, and 98% for his first four chapter exams. If the fifth exam, the final exam, counts twice as much as each of the chapter exams, is it possible for the student to get a high enough final exam score to get a 90% average for the course?

**Example 11:** A rectangular swimming pool measures 18 feet by 30 feet and is surrounded by a path of uniform width around all four sides. The perimeter of the rectangle formed by the pool and the surrounding path is 132 feet. Determine the width of the path.

**Example 12:** A Delta jet leaves Lansing International Airport and travels due east at a rate of 500 mph. A United jet takes off 15 minutes later traveling in the same direction at 650 mph. How long will it take for the United jet to catch up to the Delta jet? (Be sure to use consistent units for time!)

| | ***d*** = | ***r*** · | ***t*** |
|---|---|---|---|
| **Delta** | | | |
| **United** | | | |

**Example 13:** Sam can plow a parking lot in 45 minutes. Eric can plow the same parking lot in 30 minutes. If Sam and Eric work together, how long will it take them to clear the entire parking lot? First complete the table to find what fraction of the job can be completed in the given number of minutes for each worker and then write your equation.

| | 1 minute | 2 minutes | 5 minutes | $n$ minutes |
|---|---|---|---|---|
| Eric | | | | |
| Sam | | | | |

## 1.2 Homework Problems

1. When a number is decreased by 30% of itself, the result is 56. What is the number?
2. $5\frac{1}{4}$% of what number is 12.6?
3. 25 is what % of 80?
4. Find .25% of $240.
5. After a 20% price reduction, a cell phone sold for $77. Find the original price of the phone.
6. Find two consecutive even integers such that the sum of twice the smaller integer plus the larger is 344.
7. The length of a rectangular garden plot is 6 feet less than triple the width. If the perimeter of the field is 340 feet, what are its dimensions?
8. Trinity College currently has an enrollment of 13,300 students with a projected enrollment increase of 1000 students a year. Brown college now has 26,800 students with a projected enrollment decline of 500 students per year. Based on these projections, when will the colleges have the same enrollment?
9. Sam invested $16,000 in two different stocks. The first stock showed a gain of 12% annual interest while the second stock suffered a 5% loss. If the total annual income from both investments was $1240, how much was invested at each rate?
10. How many liters of a 7% acid solution should be added to 30 liters of a 15% solution in order to obtain a 10% solution?
11. How many liters of skim milk (0% fat) must be added to 3 liters of milk containing 3.5% butterfat in order to dilute the milk to 2% butterfat?
12. Amy scored 78%, 64%, 98%, and 88% on her first four exams. What score does she need on her fifth exam in order to have an 85% average for all five exams?
13. Scott showed improvement on his five math exams throughout the semester improving by 3% on each successive exam. If the fifth exam (final exam) counted twice as much as the first four exams and his average was 79% for all five exams, what score did he receive on his first exam?
14. The length of a rectangular tennis court is 6 feet longer than twice the width. If the perimeter of the court is 228 feet, find the dimensions of the court.
15. During a road trip, Tony drove one-third the distance that Lana drove. Mark drove 24 more miles than Lana. The total distance they drove on the trip was 346 miles. How many miles did each person drive?
16. A garden hose can fill a swimming pool in 5 days. A larger hose can fill the pool in 3 days. How long will it take to fill the pool using both hoses?
17. The Smith family drove to their vacation home in Michigan in 5 hours. The trip home took only 3 hours since they averaged 26 mph more due to light traffic. How fast did they drive each way?

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**1.2 Homework Answers:** **1**. 80 **2**. 240 **3**. 31.25% **4**. $.60 or 60¢ **5**. $96.25 **6**. 114 and 116
**7**. 44 ft by 126 ft **8**. 9 years **9**. $12,000 @ 12%; $4000 @ 5% **10**. 50 liters **11**. 2.25 liters **12**. 97%
**13**. 72% **14**. 36 ft by 78 ft **15**. Tony: 46 miles; Lana: 138 miles; Mark:162 miles
**16**. 1.875 days (45 hrs) **17**. 39 mph; 65 mph

# 1.3 Quadratic Equations

**In this section you will learn to:**

- solve quadratics equations by
    1. factoring
    2. square root property
    3. quadratic formula
    4. graphing (used mainly for checking – not considered an algebraic solution)
- solve rational equations by changing to quadratic form
- use the discriminant to find the number and type of solutions

A **quadratic equation** in $x$ is an equation that can be written in the **general form**

$$ax^2 + bx + c = 0,$$

where $a$, $b$, and $c$ are real numbers, with $a \neq 0$. A quadratic equation in $x$ is also called a **second-degree polynomial equation.**

**The Zero-Product Principle:** If the product of two algebraic expressions is zero, then at least one of the factors equal to zero.

**If $AB = 0$, then $A = 0$ or $B = 0$.**

**<u>Solving Quadratic Equations by Factoring</u>:**

**Example 1:** $9x^2 = 12x$

**<u>Steps</u>:**

1. Rearrange equation so that one side is 0. (Look for GCF.)
2. Factor. (Use sum/product idea when a = 1. If $a \neq 1$, use grouping*.)
3. Set each factor equal to 0.
4. Solve each equation.
5. Check in original equation or by graphing (observe $x$-intercepts).

*Go to "Class Pages" on math home page (www.math.msu.edu) for steps using "Grouping Method".

**Example 2:** $x^2 = 3x + 10$

**Example 3:** $(x-1)(x-4) = 10$ (Can the Zero-Product Principle be used in the first step of the solution? Explain.)

**Example 4:** $4x^2 - 13x = -3$ (Use "guess & check" or "grouping method".)

**Example 5:** $x - 2 = \frac{24}{x}$

**Example 6:** $\frac{1}{x-1} + \frac{1}{x-4} = \frac{5}{4}$

**Example 7:** When the opposite of a number is added to the *reciprocal* of the number, the result is $-\frac{15}{4}$. What is the number?

**Solving Quadratic Equations Using the Square Root Method:**

**Square Root Property:** If $a > 0$, then $x^2 = a$ has two real roots: $x = \sqrt{a}$ or $x = -\sqrt{a}$
If $a = 0$, then $x^2 = a$ has one real root: $x = \sqrt{0} = 0$

In other words, when solving an equation for a variable that involves taking the square root of both sides, you must include the $\pm$ or one of the solutions could be "lost".
(Example: If $x^2 = 4$, then $x = \pm 2$.)

**Example 8:** $3x^2 - 1 = 47$

**Example 9:** $(8q-3)^2 = 5$

**Solving Quadratic Equations Using the Quadratic Formula:**

**Quadratic Formula:** If $ax^2 + bx + c = 0$, where $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

**Example 10:** Solve and simplify: $x^2 - 9 = 4x - 13$

**Example 11:** Solve and simplify: $3x^2 = 5x - 1$

**Example 12:** Solve and simplify: $2x - 3x^2 = 21$

In the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the value of $b^2 - 4ac$ is called the **discriminant.**

**Beware:** The discriminant is **NOT** $\sqrt{b^2 - 4ac}$ **!!!**

| $b^2 - 4ac > 0$ | $b^2 - 4ac < 0$ | $b^2 - 4ac = 0$ |
|---|---|---|
| | | |
| | | |

**Example 13:** Refer to Examples 10 – 12 to complete the table below.

| **Example** | **Discriminant** | **Number and Type of Solutions** |
|---|---|---|
| **10** | | |
| **11** | | |
| **12** | | |

**Example 14 (optional):** Solve and simplify: $4x^2 + 16x = 13$ (Be sure to simplify radicals!)

**Example 15:** Applications of quadratic equations can be found in the medical field as well as many other sciences. For example, consider a piece of artery or vein which is approximately the shape of a cylinder. The velocity $v$ at which blood travels through the arteries or veins is a function of the distance $r$ of the blood from the axis of symmetry of the cylinder. Specifically, for a wide arterial capillary the following formula might apply:

$v = 1.185 - (185 \cdot 10^4)r^2$, where $r$ is measured in cm and $v$ is measured in cm/sec.

(a) Sketch a picture of the cylinder including the axis of symmetry.

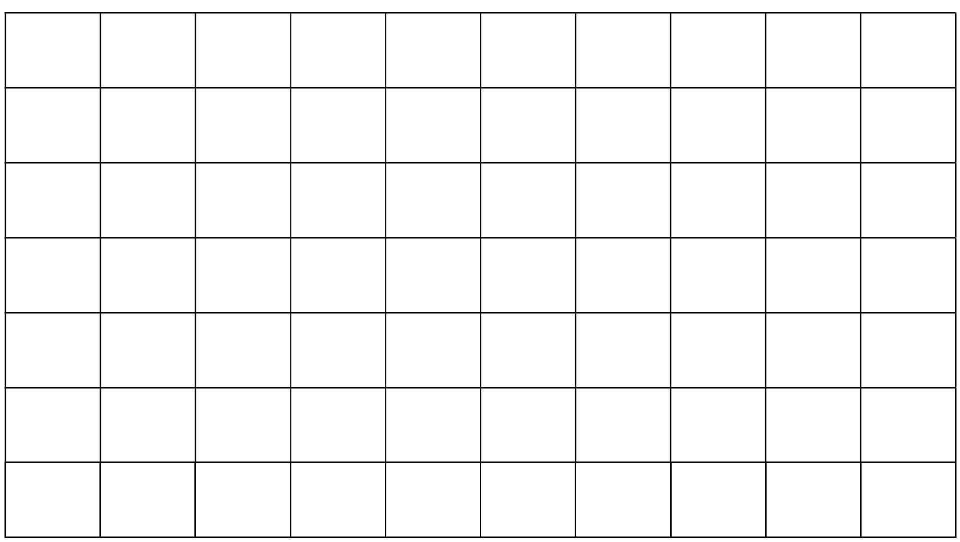
(b) Find the velocity of the blood traveling on the axis of symmetry of the capillary.

(c) Find the velocity of blood traveling $6 \cdot 10^{-4}$ cm from the axis of symmetry.

(d) According to this model, where in the capillary is the velocity of the blood 0?

(e) What are the allowable values (domain) for $r$?

(f) Sketch a graph of this equation and interpret. Be sure to label your axes.



## 1.3 Homework Problems

**Solve Problems 1-6 by factoring:**

1. $3x^2 = 5x$
2. $x^2 - 15 = 2x$
3. $-10x = x^2 + 25$
4. $a(a-12) - 15 = 30$
5. $2x^2 - 4x = 30$
6. $3m^2 = 7m + 6$
7. $\frac{1}{x} = \frac{1}{3} - \frac{1}{x+2}$
8. $\frac{1}{x-1} = 1 - \frac{2}{x+1}$

**Solve Problems 7-9 by using the quadratic formula:**

9. $x^2 + 15 = 8x$
10. $4x^2 - 8x + 1 = 0$
11. $4x^2 = 2x + 7$

**Solve Problems 10-12 using the square root method:**

12. $x^2 = \frac{25}{49}$
13. $(2x+11)^2 + 5 = 3$
14. $(3x-4)^2 = 8$
15. Solve for $t$: $h = 16t^2 - 4$
16. Solve for $x$: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

**For Problems 15-17, determine the number and type of solutions by examining the discriminant.**

17. $-3x^2 = 21 - 2x$
18. $2x^2 - 20x + 49 = 0$
19. $9x^2 + 49 = 42x$

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**1.3 Homework Answers:** **1**. $\left\{0, \frac{5}{3}\right\}$ **2**. {-3, 5} **3**. {-5} **4**. {-3, 15} **5**. {-3, 5} **6**. $\left\{-\frac{2}{3}, 3\right\}$ **7**. $\left\{2 \pm \sqrt{10}\right\}$ **8**. {0, 3} **9**. {3, 5} **10**. $\left\{\frac{2 \pm \sqrt{3}}{2}\right\}$ **11**. $\left\{\frac{1 \pm \sqrt{29}}{4}\right\}$ **12**. $\left\{\pm \frac{5}{7}\right\}$ **13**. $\phi$ **14**. $\left\{\frac{4 \pm 2\sqrt{2}}{3}\right\}$ **15**. $\left\{\pm \frac{\sqrt{h+4}}{4}\right\}$ **16**. $\left\{\pm \frac{a\sqrt{b^2 + y^2}}{b}\right\}$ **17**. 0 real roots **18**. 2 real roots **19**. 1 real root

# 1.4 Application of Quadratic Equations

**In this section you will learn to:**

- use quadratic equations to solve word problems

Recall the **Steps/Tips for Solving Word Problems** used in Section 1.2:

1. **Read** the problem carefully. **Underline** key words and phrases. Let $x$ (or any variable) represent one of the unknown quantities. **Draw** a picture or diagram if possible.

2. If necessary write expressions for **other unknowns** in terms of $x$.

3. Write a **verbal model** of the problem and then replace words with numbers, variables and/or symbols.

4. **Solve** the equation. **Answer** the question in the problem. **Label** answers!

5. **Check** your answer(s) in the original word problem (not in your equation).

**Example 1:** The length of a rectangle exceeds its width by 3 feet. If its area is 54 square feet, find its dimensions.

**Example 2:** The MSU football stadium currently has the one of the largest HD video screens of any college stadium. The rectangular screen's length is 72 feet more than its height. If the video screen has an area of 5760 square feet, find the dimensions of the screen. (MSU math fact: The area of the video screen is about 600 $ft^2$ larger than Breslin's basketball floor.)

**Example 3:** Find the dimensions of a rectangle whose area is 180 $cm^2$ and whose perimeter is 54 cm.

**Example 4:** When the sum of 8 and twice a positive number is subtracted from the square of the number, the result is 0. Find the number.

**Example 5:** In a round-robin tournament, each team is paired with every team once. The formula below models the number of games, $N$, that must be played in a tournament with $x$ teams. If 55 games were played in a round-robin tournament, how many teams were entered?

$$N = \frac{x^2 - x}{2}$$

**Example 6:** Find at least two quadratic equations whose solution set is $\left\{-\frac{2}{3}, 5\right\}$.

**Example 7:** A piece of sheet metal measuring 12 inches by 18 inches is to have four equal squares cut from its corners. If the edges are then to be folded up to make a box with a floor area of 91 square inches, find the depth of the box.

**Example 8:** The height of a toy rocket shot upward from the ground, with an initial velocity of 128 feet/second, is given by the equation $h = -16t^2 + 128t$, where the height $h$ of the object is a function of the time $t$.

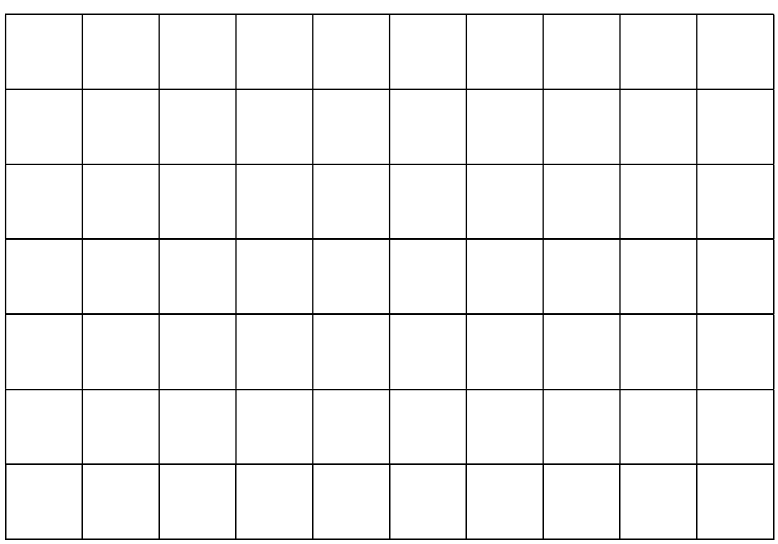
(a) How high is the rocket after 1 second?

(b) How long will it take the object to hit the ground? (Round answer to nearest hundredth.)

(c) At what time(s) will the rocket be 240 feet off of the ground?

(d) Sketch a graph of this function (height as a function of time) using the TABLE feature on your calculator. Be sure to label your axes.

(e) What is a reasonable domain for $t$?



**Example 9:** John drove his moped from Lansing to Detroit, a distance of 120 km. He drove 10 km per hour faster on the return trip, cutting one hour off of his time. How fast did he drive each way?

(Hint: Since $d = rt$ then $t = \dfrac{d}{r}$.)

| | $d$ = | $r$ · | $t$ |
|---|---|---|---|
| **Lansing → Detroit** | | | |
| **Detroit → Lansing** | | | |

**Example 10:** When tickets for a rock concert cost $15, the average attendance was 1200 people. Projections showed that for each 50¢ decrease in ticket prices, 40 more people would attend. How many attended the concert if the total revenue was $17,280?

## 1.4 Homework Problems

1. The base of a triangle exceeds its height by 17 inches. If its area is 55 square inches, find the base and height of the triangle.

2. A regulation tennis court for a doubles match is laid out so that its length is 6 feet more than two times its width. The area of the doubles court is 2808 square feet. Find the length and width of a doubles court.

3. If two opposite sides of a square are increased by 10 meters and the other sides are decreased by 8 meters, the area of the rectangle that is formed is 63 square meters. Find the area of the original square.

4. If 120 games were played in a round-robin tournament, how many teams were entered? (Refer to Example 5 in class notes for formula.)

5. A quadratic equation has two roots: ¾ and -5. (a) Find a quadratic equation where the coefficient of the $x^2$ term is 1. (b) Find a second equation that has only integers as coefficients.

6. The height of a toy rocket launched from the ground with an initial velocity of 128 feet/second, is given by the equation $h = -16t^2 + 128t$, where *h* represents the height of the rocket after *t* seconds. How long will it take the rocket to hit the ground? (Round answer to nearest hundredth.)

7. The height of an object thrown upward from the roof of a building 200 feet tall, with an initial velocity of 100 feet/second, is given by the equation $h = -16t^2 + 100t + 200$, where *h* represents the height of the object after *t* seconds. At what time(s) will the object be 300 feet above the ground? (Round answer to nearest hundredth.)

8. If the speed were increased by 10 mph, a 420-mile trip would take 1 hour less time. How long will the trip take at the slower speed?

9. Jack drove 600 miles to a convention in Washington D. C. On the return trip he was able to increase his speed by 10 mph and save 3 hours of driving time. (a) Find his rate for each direction. (b) Find his time for each direction.

10. When tickets for a water park cost $12, the average attendance was 500 people. Projections showed that for each $1 increase in ticket prices, 50 less people would attend. At what ticket price would the receipts be $5600.

11. A piece of tin, 12 inches on a side, is to have four equal squares cut from its corners. If the edges are then to be folded up to make a box with a floor area of 64 square inches, find the depth of the box.

---

**1.4 Homework Answers:** **1**. base: 22 inches; height: 5 inches **2**. 78 feet by 36 feet

**3.** 121 sq. meters **4.** 16 teams **5.** (a) $x^2 + \frac{17}{4}x - \frac{15}{4} = 0$; (b) $4x^2 + 17x - 15 = 0$ (answers vary)

**6.** 8 seconds **7.** 1.25 and 5 seconds **8.** 7 hours **9**. (a) 40 mph; 50 mph (b) 15 hours; 12 hours

**10**. $14 ($2 increase) or $8 ($4 decrease) **11.** 2 inches

# 1.6 Polynomial & Radical Equations

**In this section you will learn to:**

- solve polynomial equations using factoring
- solve radical equations
- solve equations with rational exponents
- solve equations in quadratic form

A **polynomial equation of degree *n*** is defined by

$$a_n x^n + a_{n-1} x^{n-1} + . \quad . \quad . \quad . \quad . + a_2 x^2 + a_1 x + a_0 = 0, \text{ where}$$

$n$ is a **nonnegative integer** (whole number)**,**
$a_n, a_{n-1}, a_{n-2}, \quad . \quad . \quad . a_2, a_1, a_0$ are **real numbers,**
$a_n \neq 0$,
$a_n$ is called the **leading coefficient,**
$a_0$ is called the **constant term** (Recall: $x^0 = 1$, for $x \neq 0$, therefore $a_0 x^0 = a_0$),
The **degree** of the polynomial is ***n*** (the highest degree of any term in the polynomial).

<u>**Solving Polynomials by Factoring**</u>**:**

**Example 1:** Solve: $4x^4 = 12x^2$

**Example 2:** Solve: $9y^3 + 8 = 4y + 18y^2$

**Example 3:** Solve: $x^3 - 2x^2 - 6x = 0$

## **Solving Radical Equations:**

**Example 4:** Solve: $x - \sqrt{x+11} = 1$

**Steps:**

1. Isolate radical(s).
2. Square both sides.
3. Expand.
4. Solve for x.
5. **Check answer!**

## **Solving Equations in Quadratic Form:**

**Example 5:** Solve $x - 3\sqrt{x} - 10 = 0$ using substitution.

Now solve $x - 3\sqrt{x} - 10 = 0$ using factoring.

**Example 6:** Solve $x^{-2} - 7x^{-1} - 8 = 0$ using substitution.

Now solve $x^{-2} - 7x^{-1} - 8 = 0$ using factoring.

**Example 7:** Solve $x^{\frac{2}{5}} - 2x^{\frac{1}{5}} - 3 = 0$ using substitution.

Now solve $x^{\frac{2}{5}} - 2x^{\frac{1}{5}} - 3 = 0$ using factoring.

**Example 8:** Factor each of the following using the factoring method. You do not need to solve the equation.

$x^4 - 2x^2 - 8 = 0$

$x - 2\sqrt{x} = 24$

$x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 15 = 0$

$2x^{-2} - 3x^{-1} - 5 = 0$

$(x+2)^2 - 3(x+2) - 10 = 0$

**Solving Equations with Rational Exponents:**

Recall: $x^{\frac{1}{2}} = \sqrt{x}$, $x^{\frac{1}{3}} = \sqrt[3]{x}$, … $x^{\frac{1}{n}} = \sqrt[n]{x}$ .

In general, $x^{\frac{n}{m}} = \sqrt[m]{x^n}$ or $\left(\sqrt[m]{x}\right)^n$ where $n$ is the ***n*th power** of $x$ and $m$ is the ***m*th root** of $x$.

**Example 9:** How would you enter the following rational exponents or radicals into Webwork?

$8^{\frac{1}{3}} \rightarrow$ $\sqrt[5]{2x-1} \rightarrow$ $\sqrt{x^3} \rightarrow$

**Example 10:** Simplify each of the following:

$64^{\frac{2}{3}}$ $\sqrt[4]{81}$ $\sqrt[3]{-27x^{12}}$

**Example 11:** Solve: $8x^{\frac{5}{3}} - 24 = 0$
(Hint: First isolate the exponential part of the equation.)

**Example 12:** Solve: $(x+5)^{\frac{3}{2}} = 8$

**Example 13:** For each planet in our solar system, its year is the time it take the planet to revolve once around the sun. The formula below models the number of Earth days in a planet's year, $E$, where $x$ is the average distance of the planet from the sun, in millions of kilometers. To the nearest kilometer, what is the average distance of Earth from the sun?

$$E = 0.2x^{\frac{3}{2}}$$

What is the average distance to the nearest mile? (1 mile is about 1.61 kilometers)

## 1.6 Homework Problems

**Solve each of the equations below using any appropriate method:**

1. $5x^5 - 25x^3 = 0$
2. $x^3 + 6x = 5x^2$
3. $\frac{12}{x} - \frac{x}{2} = x - 3$
4. $x^4 - 2x^2 + 1 = 0$
5. $\frac{3x}{2} - \frac{2x}{x-1} + 3 = x$
6. $\sqrt{2x-3} + 1 = x$
7. $2x - 5\sqrt{x} = -3$
8. $x^{\frac{2}{3}} + 5x^{\frac{1}{3}} - 6 = 0$

*9. $\sqrt{x+5} + \sqrt{x} - 1 = 0$

*10. $\sqrt{5-x} + \sqrt{5+x} = 4$

*11. $\sqrt{2x+3} = 1 - \sqrt{x+1}$

12. $2x^4 = 50x^2$
13. $2x^3 + 9 - 18x = x^2$
14. $3x^{\frac{3}{4}} - 24 = 0$
15. $(x-7)^{\frac{3}{2}} = 8$
16. $x^3 + 3x^2 = 2x + 6$
17. $\sqrt{x^2 + 1} = \frac{\sqrt{-7x + 11}}{\sqrt{6}}$
18. $\sqrt[3]{x+7} = 4$

*Hint: When solving equations involving two radicals, isolate one of the radicals and perform the steps given in Example 4 two times. These are extra skill development problems and will not be on Exam 1 or the final exam.

**Solve each of the following equations using substitution:**

19. $x^{-2} - x^{-1} = 20$
20. $2x - 5\sqrt{x} = -3$
21. $x^3 - 7x^{\frac{3}{2}} - 8 = 0$

22. There are approximately 88 Earth days in the year of the planet Mercury which is the closest planet to the sun. Use the formula given in Example 13 to estimate the average distance of Mercury from the sun.

---

**1.6 Homework Answers: 1**. $\{0, \pm\sqrt{5}\}$ **2**. {0, 2, 3} **3**. {-2, 4} **4**. $\{\pm 1\}$ **5**. {2, -3} **6**. {2} **7**. $\left\{\frac{9}{4}, 1\right\}$ **8**. {-216, 1} **9**. $\phi$ **10**. $\{\pm 4\}$ **11**. {-1} **12**. {-5, 0, 5} **13**. $\left\{-3, \frac{1}{2}, 3\right\}$ **14**. {16} **15**. {11} **16**. $\{-3, \pm\sqrt{2}\}$ **17**. $\left\{-\frac{5}{3}, \frac{1}{2}\right\}$ **18.** {57} **19**. $\left\{-\frac{1}{4}, \frac{1}{5}\right\}$ **20.** $\left\{\frac{9}{4}, 1\right\}$ **21**. {4}

**22.** 58,000,000 km or 36,000,000 miles

# 1.7 Inequalities

**In this section you will learn to:**

- understand the difference between intersection and union of sets
- use interval notation
- understand properties of inequality
- solve linear (and compound) inequalities
- solve polynomial inequalities
- solve rational inequalities

Recall: Compound Sentences Using "**and**" and "**or**":

| Intersection | Union |
|---|---|
| Recall: The **intersection** of two sets is the set consisting of those values common to *both* sets and generally involves the conjunction "**and**". The symbol $\cap$ is used for intersection. (Think of intersection as "overlap".) | The **union** of two sets is the set consisting of those values in either *one or both* sets and general involves the conjunction "**or**". The symbol $\cup$ is used for union. (Think of union as "uniting".) |
| $x > 4$ **and** $x < 8$ | $w > 12$ **or** $w < 3$ |
| $4 < x < 8$ ($x$ is "between" 4 and 8) | $a = 3$ **or** $a = -5$ |
| $n$ is a negative number **and** n > -5 | $n$ is a negative number **or** $n > -5$ |

**<u>Interval Notation</u>:**

| Inequality | Graph | Interval Notation |
|---|---|---|
| $x < 4$ | | |
| $x \geq -3$ | | |
| $-3 < x \leq 2$ | | |
| $x \leq -3$ or $x > 5$ | | |
| all real #'s | | |

**Example 1:** Graph each solution on a number line and then write your solution using interval notation.

*n* is a negative number **and** n > -5

*n* is a negative number **or** *n* > -5

| Properties of Inequalities | | |
|---|---|---|
| **+/- Property of Inequality** | **Multiplication/Division Property of Inequality** | |
| | **$c > 0$ (*c* is positive)** | **$c < 0$ (*c* is negative)** |
| If $a < b$, then<br><br>$a+c<b+c$<br><br>$a-c<b-c$<br><br>(Adding or subtracting does not affect the > or < sign.) | If $a < b$, then<br><br>$ac < bc$<br><br>$\frac{a}{c} < \frac{b}{c}$<br><br>(Multiplying or dividing by a positive number does not affect the > or < sign.) | If $a < b$, then<br><br>$ac > bc$<br><br>$\frac{a}{c} > \frac{b}{c}$<br><br>(When multiplying or dividing by a negative number, reverse the > or < sign.) |
| In other words, inequalities are solved using properties similar to those used for solving equations. The one exception is when *multiplying or dividing by a negative quantity*, as the inequality *symbol must then be reversed.* | | |

**Example 1:** Solve and graph the inequality below. Write the answer using interval notation.

$2-3x \le 5$

**Example 2:** Solve and graph the inequality below. Write the answer using interval notation.

$5-(7x+19) > 13(x-5)$

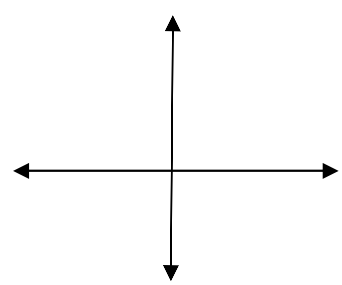
**Example 3:** Solve the compound inequality $-3 < \frac{2}{3}x + 1 \leq 5$. Write answer using interval notation. Will this inequality involve the union or intersection of sets?

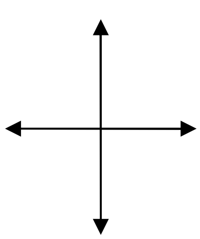
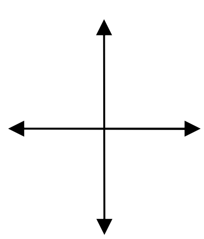
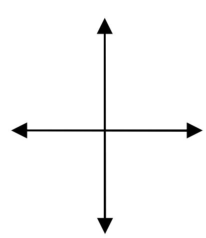
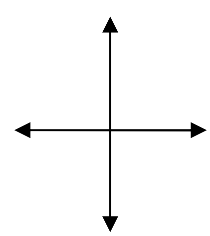
(a) Solve by isolating the variable $x$.

(b) Solve by writing each inequality separately.

**Example 4:** John is starting his new job as a Cutco sales rep. Plan A offers a base pay of \$2000/month plus a 3% commission on sales. Plan B offers a base pay of only \$500 a month but he would receive a 15% commission on all sales. What level of monthly sales is needed for John to earn more under Plan B?

Consider the graph of the polynomial equation $y = x^2 - 2x$. Since the $x$-intercepts occur when $y = 0$, the $x$-intercepts for this graph are 0 and 2. Now graph this equation showing the $x$-intercepts. Then use the graph of $y = x^2 - 2x$ to determine the solution in interval notation for each of the inequalities.



| $x^2 - 2x \geq 0$ | $x^2 - 2x > 0$ | $x^2 - 2x \leq 0$ | $x^2 - 2x < 0$ |
| --- | --- | --- | --- |
|  |  |  |  |

**Example 5:** Solve $x^2 - 6 > 5x$. Write the solution using interval notation.

**Steps for Solving Polynomial Inequalities:**

1. Express as $f(x) > 0$ or $f(x) < 0$.
   (Get 0 on right side.)

2. Set $f(x) = 0$ and solve for $x$ to get **boundary points** ($x$-intercepts)**.** Sketch a graph to verify your boundary points.

3. Determine if the boundary points are included in the answer. (Open or closed interval?)

3. Plot the boundary points on a number line to obtain i**ntervals.**

4. **Test values** within each interval and evaluate $f(x)$ for each value using substitution into the original inequality or using your calculator (table or graph).
   If $f(x) > 0$, then $f(x)$ is + for interval.
   If $f(x) < 0$, then $f(x)$ is – for interval.

5. Write the solution using interval notation. Check the solution on your calculator.

**Example 6:** Solve $x^3 + x^2 - 9 \le 9x$. Write the solution using interval notation.

**Example 7:** Solve $x^3 + 2x^2 \geq 4x + 8$. Write the solution using interval notation.

A **rational inequality** is any inequality of the form:

$f(x) < 0$ (graph is below the *x*-axis)

$f(x) \leq 0$ (graph is on or below the *x*-axis)

$f(x) > 0$ (graph is above the *x*-axis)

$f(x) \geq 0$ (graph is on or above the *x*-axis)

where *f* is a **rational function**. ( $f(x) = \dfrac{p(x)}{q(x)}$, where *p* and *q* are polynomials and $q(x) \neq 0$ ).

**Example 8:** Solve $\dfrac{x-2}{x+5} \geq 0$.

<u>**Steps for Solving Rational Inequalities:**</u>

1. Express as $f(x) > 0$ or $f(x) < 0$. (Get "0" on right side.)

*2. Find values that make the numerator & the denominator = 0. These are the **boundary points.**

3. Determine if the boundary points are included in the answer. (Be sure to consider the points that must be excluded from the domain.)

4. Plot the boundary points on a number line to obtain i**ntervals.**

5. **Test values** within each interval and evaluate $f(x)$ for each value using substitution into the original inequality or using your calculator (table or graph).
   If $f(x) > 0$, then $f(x)$ is + for interval.
   If $f(x) < 0$, then $f(x)$ is – for interval.

6. Write the solution using interval notation. Check the solution on your calculator.

**Example 9:** Solve $\frac{x}{x+2} \geq 2$. Can you multiply both sides by $x + 2$ (LCD)? Explain.

**Example 10:** Solve $\frac{6}{x} \geq 2$.

**Example 11:** A ball is thrown vertically from a rooftop 240 feet high with an initial velocity of 64 feet per second. During which time period will the ball's height exceed that of the rooftop? (Use $h(t) = -16t^2 + v_0 t + s_0$ where $v_0$ = initial velocity, $s_0$ = initial height/position, and t = time. (You may also want to graph this function on your calculator using the viewing rectangle [0, 10, 1] by [-100, 500, 100]).

## 1.7 Homework Problems

**Solve each of the inequalities below and write the answer using interval notation.**

1. $3+5x \le 2(1+3x)$
2. $3(x+2) \le 4(x+5)$
3. $\dfrac{3(x+3)}{2} < \dfrac{2(x+7)}{3}$
4. $\dfrac{2}{3}x - x \le -\dfrac{3}{2}(x-5)$
5. $\dfrac{1}{4}x + \dfrac{2}{3}x - x > \dfrac{1}{2} + \dfrac{1}{2}(x+1)$
6. $2+x < 3x-2 \le 5x+2$
7. $0 \le \dfrac{3+x}{2} < 4$
8. $x^2 - 2x - 8 > 0$
9. $2x^2 + x - 3 \le 0$
10. $x^3 + x^2 \le 4x + 4$
11. $x^3 \ge 9x^2$
12. $x^3 - 2x^2 - 4x + 8 \le 0$
13. $9x^2 - 6x + 1 < 0$
14. $\dfrac{2}{x} < 4$
15. $\dfrac{x-4}{x+3} > 0$
16. $\dfrac{4-2x}{3x+4} \le 0$
17. $\dfrac{-x+2}{x-4} \ge 0$
18. $\dfrac{x+1}{x+3} \le 2$
19. $\dfrac{3}{x-2} \ge 5$
20. $\dfrac{x}{2x-1} - 1 \ge 0$
21. Suppose $W$ is the set of numbers less than or equal to 5, and $X$ is the set of numbers greater than 5 and less than 10. Write each of the following using interval notation.

    (a) $W$ (b) $X$ (c) $W \cup X$ (d) $W \cap X$

22. Jane is a highly motivated MTH 103 student. She earned scores of 76%, 82%, 84%, and 90% on her first four exams. At the end of the semester, her top five quiz scores totaled 88% and her Webwork average was 98%. If the final exam counts twice as much as one of her four exams, what score must she earn on the final exam if she wants a 3.5 for the course. (The range for a 3.5 is 85 – 89%.) Hint: Count the final exam as two regular exam scores. Is it still possible for Jane to earn a 4.0 for the course?

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**1.7 Homework Answers:** **1**. $[1, \infty)$ **2**. $[-14, \infty)$ **3**. $\left(-\infty, \dfrac{1}{5}\right)$ **4**. $\left(-\infty, \dfrac{45}{7}\right]$ **5**. $\left(-\infty, -\dfrac{12}{7}\right)$ **6**. $(2, \infty)$ **7**. [-3, 5) **8**. $(-\infty, -2) \cup (4, \infty)$ **9**. $\left[-\dfrac{3}{2}, 1\right]$ **10**. $(-\infty, -2] \cup [-1, 2]$ **11**. $\{0\} \cup [9, \infty)$ **12**. $(-\infty, -2] \cup \{2\}$ **13**. $\phi$ **14**. $(-\infty, 0) \cup \left(\dfrac{1}{2}, \infty\right)$ **15**. $(-\infty, -3) \cup (4, \infty)$ 16. $\left(-\infty, -\dfrac{4}{3}\right) \cup [2, \infty)$ **17**. [2,4) **18**. $(-\infty, -5] \cup (-3, \infty)$ **19**. $\left(2, \dfrac{13}{5}\right]$ **20**. $\left(\dfrac{1}{2}, 1\right]$ **21.** (a) $(-\infty, 5]$ (b) (5, 10) (c) $(-\infty, 10)$ (d) { } or $\phi$ **22.** [162, 194] or [81%, 97%]; yes, if grades are rounded up

# 1.8 Absolute Value

**In this section you will learn to:**

- solve absolute value equations
- solve absolute value inequalities
- apply absolute inequalities

The **absolute value** of a real number $x$ is denoted by $|x|$, and is defined as follows:

If $x \geq 0$ (non-negative number), then $|x| = x$.

If $x < 0$ (negative number), then $|x| = -x$.

**Absolute Value Equations:** If $k \geq 0$ and $|x| = k$, then $x = k$ or $x = -k$. For example: $|k| = 5$, then $k = \pm 5$.

Also, $\sqrt{x^2} = |x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$ For example: $\sqrt{5^2} = |5| = 5$ and $\sqrt{(-5)^2} = |-5| = 5$

**<u>Solving Absolute Value Equations</u>:** (Hint: Be sure to isolate the absolute value part first!)

**Example 1:** Solve: $|2x+3| = 7$

**Example 2:** Solve: $-2\left|3 - \frac{x}{5}\right| - 1 = -3$

**Example 3:** Solve: $|x+1| + 6 = 2$

**Example 4:** Solve: $(x+5)^{\frac{2}{3}} = 4$

**Example 5:** Solve: $\left|x^2 - 3x - 11\right| - 3 = 4$

**Example 6:** Solve (using guess and check) and graph each of the following:

$|x| = 2$ $\qquad$ $|x| > 2$ $\qquad$ $|x| < 2$

Which inequality involves the idea of "and" (intersection)? Which involves the idea of "or" (union)?

**Absolute Value Inequalities:** If $k > 0$, then $|x| \geq k$ is equivalent to $x \geq k$ **or** $x \leq -k$ (union).
If $k > 0$, then $|x| \leq k$ is equivalent to $x \leq k$ **and** $x \geq -k$ (intersection).

**<u>Solving Absolute Value Inequalities</u>:**

**Example 7:** Solve and graph the inequality below. Write answer using **interval notation.**

$|5x - 4| < 3$

**Example 8:** Solve and graph the inequality below. Write answer using **interval notation.**

$5 \leq \left|\frac{x}{2} + 3\right|$

**Example 9:** Solve: $-3|x-1|+2 \le 8$

**Example 10:** Solve: $0 < |x+2| \le 5$

**Example 11:** The average temperature in Cancun is $78°$F with fluctuations of approximately 7 degrees. Represent this temperature range using absolute value notation and then solve the inequality to find the highest and lowest temperatures in Cancun using interval notation. Also represent this fluctuation using a number line.

**Example 12:** A weight attached to a spring hangs at rest a distance of $x$ inches off of the ground. If the weight is pulled down (stretched) a distance of $L$ inches and released, the weight begins to bounce and its distance $d$ off of the ground at any time satisfies the inequality $|d-x| \le L$. If $x$ equals 4 inches and the spring is stretched 3 inches and released, solve the inequality to find the range of distances from the ground the weight will oscillate. Draw a picture to represent this situation.

# 1.8 Homework Problems:

**Solve each of the absolute value equations below:**

1. $|x-5|+8=12$
2. $-2|x+4|+3=-1$
3. $\left|\frac{2}{3}x+\frac{5}{6}\right|-\frac{7}{12}=\frac{11}{12}$
4. $-5|2m-7|+2=-13$
5. $|x^2-2x-25|=10$

**Solve each inequality. Write the solution using interval notation.**

6. $3|x+4|+5<8$
7. $-3|x-5|>-12$
8. $\frac{|3x+2|}{-4}\le -1$
9. $-3\le -2\left|3-\frac{x}{5}\right|-1$
10. $|4-3x|+12<7$
11. $-1>\frac{|2x-3|}{-3}$
12. A Steinway piano should be placed in room where the relative humidity $h$ is between 38% and 72%. Express this range with an inequality containing an absolute value.
13. The optimal depths $d$ (in feet) for catching a certain type of fish satisfy the inequality $28|d-350|-1400<0$. Find the range of depths that offer the best fishing.
14. Monthly rainfall received in Omaha, Nebraska, rarely varies by more than 1.7 inches from an average of 2.5 inches /month. Use this information to write an absolute value inequality model, then solve the inequality to find the highest and lowest amounts of monthly rainfall for this city.

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**1.8 Homework Answers:** **1**. {1, 9} **2**. {-6, -2} **3**. {-3.5, 1} **4**. {2, 5} **5**. {-5, -3, 5, 7} **6**. (-5, -3)

**7**. (1, 9) **8**. $(-\infty,-2]\cup\left[\frac{2}{3},\infty\right)$ **9**. [10, 20] **10**. $\phi$ **11**. $(-\infty,0)\cup(3,\infty)$ **12**. $|h-55|<17$

**13**. (300,400) **14.** $|r-2.5|\le 1.7$; highest: 4.2 inches; lowest: 0.8 inches