

NAME: \_\_\_\_\_

**Math 153H-01**

**Quiz 9**

**April 7, 2016**

No calculators, no notes, no books. Only pens, pencils and erasers are allowed.

1. Find  $\lim_{x \rightarrow 1} \frac{\ln x}{e^x - 1}$ .

Solution: The function  $f(x) = \frac{\ln x}{e^x - 1}$  is well-defined and continuous at the point 1, so one should just plug in the value 1 in this function in order to get the limit, which is  $f(1) = \frac{0}{e-1} = 0$ .

2. Say for each of the following integrals if it is convergent or divergent, and if it is convergent, calculate its limit:

$$\int_0^1 \frac{1}{x^2} dx$$

Solution: This integral is by definition  $\lim_{a \rightarrow 0^+} \int_a^1 \frac{dx}{x^2} = \lim_{a \rightarrow 0^+} \left(-\frac{1}{x} \Big|_a^1\right) = \lim_{a \rightarrow 0^+} \left(\frac{1}{a} - 1\right) = \infty$ . Therefore this integral is divergent.

$$\int_1^\infty \frac{1}{x^2} dx$$

Solution: This integral is by definition  $\lim_{b \rightarrow \infty} \int_1^b \frac{dx}{x^2} = \lim_{b \rightarrow \infty} \left(-\frac{1}{x} \Big|_1^b\right) = \lim_{b \rightarrow \infty} \left(1 - \frac{1}{b}\right) = 1$ . Therefore this integral is convergent.