NAME:____

Math 153H-01 **Quiz 8** **March 31, 2016**

No calculators, no notes, no books. Only pens, pencils and erasers are allowed.

1. Find the length of the curve (given in polar coordinates) $r = 1 - \cos\theta$ on $[0, 2\pi]$.
Solution: The length of the curve is given by the formula
$\int_0^{2\pi} \sqrt{r^2 + (r')^2} d\theta = \int_0^{2\pi} \sqrt{1 - 2\cos\theta + \cos^2\theta + \sin^2\theta} d\theta = \int_0^{2\pi} \sqrt{2 - 2\cos\theta} d\theta$. We can use the identity $\cos(\theta) = 1 - 2\sin^2(\frac{\theta}{2})$ and then $2 - 2\cos(\theta) = 4\sin^2(\frac{\theta}{2})$. For $0 \leq theta \leq 2\pi$, we have $0 \leq \frac{theta}{2} \leq \pi$, and over that interval $\sin(\frac{\theta}{2}) \geq 0$, and so $\sqrt{2 - 2\cos(\theta)} = 2\sin(\frac{\theta}{2})$. Consequently the length is $\int_0^{2\pi} 2\sin(\frac{\theta}{2}) d\theta = -4\cos(\frac{\theta}{2})|_0^{2\pi} = 8$.

2. Find $\lim_{x\to\infty}(\sqrt{x^2+1} - x)$.
Solution: $\sqrt{x^2+1} - x = \sqrt{x^2+1} - x \cdot \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x} = \frac{x^2+1-x^2}{\sqrt{x^2+1}+x} = \frac{1}{\sqrt{x^2+1}+x}$. As x approaches infinity, $\frac{1}{\sqrt{x^2+1}+x}$ approaches 0, and so the limit is 0.