NAME:____

Math 153H-01 **Quiz 7** **March 17, 2016**

No calculators, no notes, no books. Only pens, pencils and erasers are allowed.

1. Calculate $\int \frac{dx}{1-x^4}$.

Solution: $1 = \frac{1}{2}(1 - x^2 + 1 + x^2)$ and so $\int \frac{dx}{1-x^4} = \frac{1}{2}\int \frac{1+x^2}{1-x^4}dx + \frac{1}{2}\int \frac{1-x^2}{1-x^4}dx = \frac{1}{2}\int \frac{dx}{1-x^2} + \frac{1}{2}\int \frac{dx}{1+x^2} = \frac{1}{2}(\frac{1}{2}\int \frac{1+x}{1-x^2}dx + \frac{1}{2}\int \frac{1-x}{1-x^2}dx) + \frac{1}{2}\int \frac{dx}{1+x^2} = \frac{1}{4}\int \frac{dx}{1-x} + \frac{1}{4}\int \frac{dx}{1+x} + \frac{1}{2}\int \frac{dx}{1+x^2} = -\frac{1}{4}\ln|1-x| + \frac{1}{4}\ln|1+x| + \frac{1}{2}\tan^{-1}x + C = \frac{1}{4}\ln|\frac{1+x}{1-x}| + \frac{1}{2}\tan^{-1}x + C$

2. A pump is to empty a conical tank full of water of height 10 meters whose radius on top is 3 meters and move the water into a reservoir which is located at the same level as the top of the tank. How much work is required for the job? Assume in your answer that the force of gravity acting on a mass m is gm where $g = 10\frac{m}{s^2}$. Recall that the density of water is $1000\frac{kg}{m^3}$.

Solution: We fix the x-axis to the be axis going upward, with 0 set at the bottom of the tank and 10 at the top. The cross section at any given point x is a circle of radius $\frac{3}{10}x$. The area of this cross section is $\frac{9}{100}\pi x^2$. The volume of width dx around that cross section is of volume $\frac{9}{100}\pi x^2 dx$. Its mass is $\frac{9}{100}\pi x^2 dx \cdot 1000 = 90\pi x^2 dx$. The force of gravity acting on this mass is $90\pi x^2 dx \cdot 10 = 900\pi x^2 dx$. The work needed to pump this slice to the top of the tank is $900pix^2dx \cdot (10 - x) = \pi(9000x^2 - 900x^3)dx$. The total work needed is therefore the integral $\int_0^1 0\pi(9000x^2 - 900x^3)dx = \pi(3000x^3 - 225x^4)|_0^1 0 = pi \cdot 750000$ in Joules.