

NAME: _____

Math 153H-01

Quiz 5

February 25, 2016

No calculators, no notes, no books. Only pens, pencils and erasers are allowed.

1. Solve $y' = \cos^2(x) - \tan(x)y$, $y(0) = 1$.

Solution: $P(x) = -\tan(x)$. $\int -\tan(x)dx = \int \frac{-\sin(x)dx}{\cos x}$. Substitute $u = \cos x$ and then $\int \frac{-\sin(x)dx}{\cos x} = \int \frac{du}{u} = \ln |u| = \ln |\cos x| + C$. Define $F(x) = \ln(\cos(x))$. Then $F'(x) = P(x)$. Write $w = ye^{-F(x)} = ye^{-\ln(\cos(x))} = \frac{y}{\cos(x)}$. Then $w' = \frac{y'}{\cos x} - y \tan x \frac{1}{\cos x} = \cos x$, and so $w = \sin x + C$, hence $y = \cos x \sin x + C \cos x$. Now $y(0) = 1 = \cos 0 \sin 0 + C \cos 0 = 0 + C$ and so $C = 1$ and the final answer is $y = \cos x \sin x + \cos x$.

2. Compute the volume of a ball of radius r by the slice method.

Solution: we locate the center of the ball in the three-dimensional space at the origin and we consider slices of the ball cut perpendicularly to the x -axis. At each point x between $-r$ and r , the slice is a circle of radius $\sqrt{r^2 - x^2}$, and so the area of the slice is $\pi(r^2 - x^2)$. The volume is therefore $\int_{-r}^r \pi(r^2 - x^2)dx = \pi(r^2x - \frac{x^3}{3})|_{-r}^r = \frac{4}{3}\pi r^3$.