

NAME: \_\_\_\_\_

**Math 153H-01**

**Quiz 3**

**February 4, 2016**

No calculators, no notes, no books. Only pens, pencils and erasers are allowed.

**1.** A population of 10,000 reached 640,000 in a hundred years. How long will it take the population to double its size (assuming it follows the mathematical model of natural growth)?

Solution: The initial population was  $A = 10,000$ . The growth function is  $f(t) = 10,000 \cdot e^{\gamma t}$  for some  $\gamma$ . Now  $f(100) = 10,000 \cdot e^{100\gamma} = 640,000$ , so  $e^{100\gamma} = 64$  which means  $\gamma = \frac{1}{100} \ln(64)$ . Therefore the growth function is  $f(t) = 10,000 \cdot e^{\frac{1}{100} \ln(64)t} = 10,000 \cdot 64^{\frac{t}{100}}$ . [Recall that  $2^6 = 64$ .] The population doubles its size at time  $t$  which satisfies  $2 = 64^{\frac{t}{100}}$ , i.e.  $\frac{1}{6} = \frac{t}{100}$ , which means  $t = \frac{100}{6} = 16\frac{2}{3}$ .

**2.** Solve the system  $y'' = -4y$ ,  $y(0) = 3$ ,  $y'(0) = 6$ .

Solution: the general solution to  $y'' = -4y$  is  $y = A \cos(2t) + B \sin(2t)$ .  $y(0) = A = 3$  and  $y'(0) = 2B = 6$  so  $B = 3$ . Therefore the solution is  $y = 3 \cos(2t) + 3 \sin(2t)$ .