NAME:____________________________________

Math 153H-01 **Quiz 2** **January 28, 2016**

No calculators, no notes, no books. Only pens, pencils and erasers are allowed.

1. Calculate $\int e^x \sin x dx$.
Solution: Write $f(x) = \sin x$ and $g(x) = e^x$. Then $f'(x) = \cos x$ and $g'(x)$. We use integration by parts $\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$:

$$\int e^x \sin x dx = e^x \sin x - \int e^x \cos x dx.$$

We shall do it again for $\int e^x \cos x dx$ with $F(x) = \cos x$, $G(x) = e^x$:

$$\int e^x \cos x = e^x \cos x - \int e^x(-\sin x)dx = e^x \cos x + \int e^x \sin x dx.$$

Plug this in the previous expression and get

$$\int e^x \sin x = e^x \sin x - e^x \cos x - \int e^x \sin x dx.$$

Therefore $2\int e^x \sin x = e^x(\sin x - \cos x) + C$, and so $\int e^x \sin x dx = \frac{1}{2}e^x(\sin x - \cos x) + C$.

2. Calculate $\int \arccos x dx$.

Solution 1: Write $f(x) = \arccos x$, $g(x) = x$. Then $f'(x) = -\frac{1}{\sqrt{1-x^2}}$ and $g'(x) = 1$. By integration by parts:

$$\int \arccos x dx = x \arccos x + \int \frac{x}{\sqrt{1-x^2}} dx.$$

The integral on the right-hand side can be calculated using the substitution $u = 1 - x^2$. In this case we get

$$\int \frac{x}{\sqrt{1-x^2}} dx = \int \frac{-du}{2\sqrt{u}} = -\sqrt{u} + C = -\sqrt{1-x^2} + C.$$

Therefore

$$\int \arccos x dx = x \arccos x - \sqrt{1-x^2} + C.$$

Solution 2: Write $y = \arccos x$. Then $\cos y = x$. Note that $0 \le y \le \pi$, over which $\sin y$ is positive and satisfies $\sin y = \sqrt{1 - \cos^2 y} = \sqrt{1-x^2}$. Use the identity $\int y dx = xy - \int x dy$:

$$\int \arccos x dx = x \arccos x - \int \cos y dy = x \arccos z - \sin y + C = x \arccos x - \sqrt{1-x^2} + C.$$