NAME:______________________________________

Math 153H-01 **Quiz 1** **January 21, 2016**

No calculators, no notes, no books. Only pens, pencils and erasers are allowed.

1. Calculate $\int \sqrt{4-x^2}dx$. [Hint: trig identities]
Substitute $x = 2\sin\theta$ where $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. Then, using the identity $\cos^2 x + \sin^2 x = 1$, we have $4 - x^2 = 4 - 4\sin^2\theta = 4\cos^2\theta$. Over that interval $\cos\theta \geq 0$, so $\sqrt{4\cos^2\theta} = 2\cos\theta$. Now $\frac{dx}{d\theta} = 2\cos\theta$. The integral thus becomes $\int 4\cos^2\theta d\theta$. Now, by using the trig identity $\cos(2x) = 2\cos^2 x - 1$ the integral becomes

$$\int (2\cos(2\theta) + 2)d\theta = \sin(2\theta) + 2\theta + C.$$

We want to express the solution in terms of x. For that we use the identities $\sin(2x) = 2\sin x\cos x$ and $\cos x = \sqrt{1-\sin^2 x}$ (which holds because $\cos x$ is non-negative). Note that $\frac{x}{2} = \sin x$. The solution then becomes

$$= 2\sin(\theta)\cos(\theta) + 2\theta + C = x\sqrt{1-\frac{x^2}{4}} + 2\arcsin\frac{x}{2} + C.$$

2. Evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x\cos x dx$.
Substitute $u = \sin x$. Then $\frac{du}{dx} = \cos x$. The endpoints are $\sin 0 = 0$ and $\sin\frac{\pi}{2} = 1$. Therefore $\int_0^{\frac{\pi}{2}} \sin^2 x\cos x dx = \int_0^1 u^2 du = \frac{u^3}{3}|_0^1 = \frac{1}{3}$.