NAME:_____

Math 153H-01 **Quiz 10** **April 14, 2016**

No calculators, no notes, no books. Only pens, pencils and erasers are allowed.

1. Find $\lim_{n\to\infty}(1+\frac{2}{n})^{\frac{1}{n}}$.
Solution: $(1+\frac{2}{n})^{\frac{1}{n}} = e^{\frac{1}{n}\ln(1+\frac{2}{n})}$. $\lim_{n\to\infty}\frac{1}{n}\ln(1+\frac{2}{n}) = 0\cdot 0 = 0$, so $\lim_{n\to\infty} e^{\frac{1}{n}\ln(1+\frac{2}{n})} = 1$.

2. Say whether $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ is convergent or divergent. Explain why. If it is convergent, find its sum. [Hint: telescopic series.]
Solution: Notice that $\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n-1}$. Write $S_k = \sum_{n=1}^{k}\frac{1}{n(n+1)}$, and so $S_k = 1 - \frac{2}{+}\frac{2}{-}\frac{1}{3} + \cdots - \frac{1}{k+1} = 1 - \frac{1}{k+1}$. Therefore $\lim_{k\to\infty}(S_k) = 1$, which means that $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ is convergent and its sum is 1.