Convergent Series.
A series $\sum_{n=1}^{\infty} a_n$ is convergent if the limit $L = \lim_{k\to\infty} S_k$ exists and is finite where $S_k = \sum_{n=1}^{k} a_n$. In that case, we say that L is the sum of $\sum_{n=1}^{\infty} a_n$ and write $\sum_{n=1}^{\infty} a_n = L$. If the series is not convergent, we say it is divergent.

Absolutely Convergent Series.
If $\sum_{n=1}^{\infty} |a_n|$ is convergent, we say that $\sum_{n=1}^{\infty} a_n$ is absolutely convergent. Every absolutely convergent series is also convergent. If a series is convergent but not absolutely convergent, we say it is conditionally convergent.

nth Term Test.
If $\sum_{n=1}^{\infty} a_n$ is convergent then $\lim_{n\to\infty} a_n$ must be 0. Therefore, if $\lim_{n\to\infty} a_n$ is not 0 then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

Comparison Test.
Suppose $|a_n| \le b_n$ for any n. If $\sum_{n=1}^{\infty} b_n$ is convergent then $\sum_{n=1}^{\infty} a_n$ is absolutely convergent. Therefore, if $\sum_{n=1}^{\infty} a_n$ is divergent then so is $\sum_{n=1}^{\infty} b_n$.

Ratio-Comparison Test.
Suppose for any n and $\lim_{n\to\infty} \frac{|a_n|}{b_n}$ exists and is finite. If $\sum_{n=1}^{\infty} b_n$ is convergent then $\sum_{n=1}^{\infty} a_n$ is absolutely convergent. Therefore, if $\sum_{n=1}^{\infty} a_n$ is divergent, so is $\sum_{n=1}^{\infty} b_n$.

Alternating Series Test.
Suppose a_n is an alternating sequence whose absolute values are decreasing (or at least non-increasing) and $\lim_{n\to\infty} a_n = 0$. Then $\sum_{n=1}^{\infty} a_n$ is convergent.

Integral Test.
Suppose $f(x)$ is a positive decreasing function on $[1, \infty)$. Let a_n be the sequence $a_n = f(n)$ for any integer n. Then $\sum_{n=1}^{\infty} a_n$ converges if and only if the improper integral $\int_1^{\infty} f(x)dx$ converges.

Ratio Test.
Suppose $\lim_{n\to\infty} |\frac{a_n}{a_{n-1}}| = L$ exists. If $L < 1$ then $\sum_{n=1}^{\infty} a_n$ is convergent. If $L > 1$ then $\sum_{n=1}^{\infty} a_n$ is divergent.

Root Test.
Suppose $\lim_{n\to\infty} |a_n|^{\frac{1}{n}} = L$ exists. If $L < 1$ then $\sum_{n=1}^{\infty} a_n$ is convergent. If $L > 1$ then $\sum_{n=1}^{\infty} a_n$ is divergent.

Telescopic Series.
Suppose $a_n = b_n - b_{n+1}$ for every n and $\lim_{n\to\infty} b_n = L$ exists and is finite. Write $S_k = \sum_{n=1}^{k} a_n$. Then $S_k = b_1 - b_{k+1}$. Therefore $\lim_{k\to\infty} S_k = \lim_{n\to\infty}(b_1 - b_{k+1}) = b_1 - L$. Consequently $\sum_{n=1}^{\infty} a_n$ is convergent and its sum is $b_1 - L$.

Geometric Series. Consider $a_n = cq^n$ for some constants c and q where $q \neq 1$. Write $S_k = \sum_{n=1}^{k} a_n$. Then $S_k + a_{k+1} = S_{k+1} = \sum_{n=1}^{k+1} a_n = a_1 + qS_k$. Therefore $S_k = \frac{a_1 - a_{k+1}}{1-q} = \frac{c(q-q^{k+1})}{1-q}$. Consequently the series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if $|q| < 1$. In this case, the sum is $\frac{cq}{1-q} = \frac{a_1}{1-q}$.