**Linear Inequalities:**

Basic Rules for Inequalities:

- If $a \leq b$ then $a + c \leq b + c$ for any $c$. (You can add the same number to both sides of an inequality)

- If $a \leq b$ and if $c > 0$ then $ac \leq bc$. (If you multiply both sides by a positive number, the inequality remains pointing the same way.)

- If $a \leq b$ and if $c < 0$ then $ac \geq bc$. (If you multiply both sides by a negative number, the direction of the inequality gets reversed.)

Recall that $a > b$ and $b < a$ mean the same thing: The inequality sign points at the smaller number. Recall also that $a \leq b$ means $a < b$ or $a = b$. Rules similar to the above hold for strict inequalities:

- If $a < b$ then $a + c < b + c$ for any $c$. (You can add the same number to both sides of an inequality)

- If $a < b$ and if $c > 0$ then $ac < bc$ (If you multiply both sides by a positive number, the inequality remains pointing the same way.)

- If $a < b$ and if $c < 0$ then $ac > bc$ (If you multiply both sides by a negative number, the direction of the inequality gets reversed.)

First example: Graph

\[ x + y \leq 1 \]

First get $y$ on one side by adding $-x$ to both sides of the inequality:

\[ y \leq 1 - x. \]
Next sketch $y = 1 - x$.

Notice that if $x = 0$ then $y = 1$ and if $y = 0$ then $x = 1$ so $(1,0)$ and $(0,1)$ are two points on the line. Thus we sketch the line between those points. Then note that if $x$ is any point, the point on the line $y = 1 - x$ directly above or below $x$ gives the value of $y$ such that $y = 1 - x$. Any point on the same vertical line below that point will have coordinates $(x, y)$ with $y < 1 - x$, so we shade the region below the line $y = 1 - x$.

Next example: Sketch $2x - 3y < 6$

First add $3y$ to both sides; $2x < 6 + 3y$. Then add $-6$ to both sides:

$2x - 6 < 3y$.

Then multiply both sides by $1/3$. Since $1/3$ is positive, the inequality remains pointing in the same direction:

$(2/3)x - 2 < y$.

Next sketch $y = (2/3)x - 2$. Notice that if $x = 0$ then $y = -2$ and if $x = 3$ then $y = 0$ so $(0, -2)$ and $(3, 0)$ are two convenient points on the line. Sketch the line through those two points. For any $x$, the point $(x, y)$ where the vertical line through $x$ meets the line gives the points where $y = (2/3)x - 2$. Any point $(x, y)$ on the vertical line through $x$ above the line $y = (2/3)x - 2$ satisfies $y > (2/3)x - 2$. Thus we shade in the region above the line $y = (2/3)x - 2$. We sketch the line $y = (2/3)x - 2$ as a dotted line to
indicate that the points on the line are not included.

Next example: Sketch

\[ x \leq 2 \]

There is no \( y \) involved. First sketch the vertical line \( x = 2 \). Notice that any point on the line or to left of that line satisfy \( x \leq 2 \), so we sketch the entire region to the left of the line \( x = 2 \).
The points \((x, y)\) that satisfy \(x \geq 0\) are all points on the y-axis or to the right of it, in other words, the right-hand plane. The points \((x, y)\) that satisfy \(y \geq 0\) are the points on the x-axis or above it. The points that satisfy both conditions is the first quadrant.

Next example: Sketch

\[
\begin{align*}
x & \geq 0 \text{ and } y \geq 0
\end{align*}
\]

This means find the region where the points \((x, y)\) satisfy both inequalities.

If we sketch the points \((x, y)\) that satisfy the first equation \(x + y \geq 3\), i.e., \(y \geq 3 - x\). We get all points on or above the line \(y = 3 - x\).

Then sketch the points \(2y \leq x\), i.e., \(y \leq (1/2)x\), by sketching the points on or below the line \(y = (1/2)x\).
Then shade the region common to both of the above regions:

Next example: Sketch

\[
\begin{align*}
x + y &\geq 3 \\
-x + 2y &\leq 0 \\
x &\geq 0 \\
y &\geq 0.
\end{align*}
\]

We already know what the region where \((x,y)\) satisfy the first two equations. We also know that the region where \((x,y)\) satisfy the last two equations is the first quadrant. The points \((x,y)\) that satisfy all four equations is then the part of the previously found region that is in the first quadrant.
Next example: Sketch

\[
\begin{align*}
    x + y & \geq 3 \\
    -x + 2y & \leq 0 \\
    x & \geq 0 \\
    y & \geq 0 \\
    x & \leq 5
\end{align*}
\]

The points in the plane \((x, y)\) that satisfy \(x \leq 5\) are those points on or to the left of the vertical line \(x = 5\). Hence the points that satisfy the first four equations, which we found in the previous example, which are on or to the left of the line \(x = 5\), will satisfy all the above equations:
Last example: Sketch

\[
\begin{align*}
  x + y & \leq 150 \\
  30x + 15y & \leq 3000 \\
  x & \leq 60 \\
  x & \geq 0 \\
  y & \geq 0
\end{align*}
\]

Notice the second equation simplifies to \(2x + y \leq 200\).

First sketch all points on or below \(y = 150 - x\), then all points on or below \(y = 200 - 2x\). By solving both equations, we find the point of intersection is \((50, 100)\). Then restrict the region to the first quadrant since \(x \geq 0\) and \(y \geq 0\). Then find all points in that region that are on or to the left of the vertical line \(x = 60\):