Math 360 Chapter 4 Questions

Bond Pricing
Ex 4.1.2

• A 6 % bond maturing in 8 years with semiannual coupons to yield 5 % convertible semiannually is to be replaced by a 5.5 % bond yielding the same return. In how many years should the new bond mature? (Both bonds have the same price, yield rate, and face amount)
Ex 4.1.2 soln

\[
\frac{F}{(1.025)^{16}} + F \cdot (0.03) \cdot a_{16|0.025} = \frac{F}{(1.025)^{2n}} + F \cdot (0.0275) \cdot a_{2n|0.025}
\]

\[
= \frac{F}{(1.025)^{2n}} + F \cdot (0.0275) \cdot \left(1 - \frac{1}{(1.025)^{2n}}\right)
\]

\[
= F \cdot \left(\frac{1}{(1.025)^{2n}} + 1.1 \cdot \left(1 - \frac{1}{(1.025)^{2n}}\right)\right) \Rightarrow \frac{1}{(1.025)^{2n}} = 0.347250
\]

\[\Rightarrow n = 21.5\]
Ex 4.1.7

- Two semi-annual bonds, each of face amount 100, are offered for sale at a combined price of 240. Both bonds have the same term to maturity, but the coupon rate for one is twice that of the other. The difference in price of the two bonds is 24. Prices are based on a nominal annual yield rate of 3 %. Find the coupon rates of the two bonds.
Ex. 4.1.7 soln

\[ P_1 + P_2 = 240 \]
\[ P_1 - P_2 = 24 \]
\[ \Rightarrow P_1 = 132, P_2 = 108 \]
\[ r_1 = 2 \cdot r_2 \]
\[ P_1 = 132 = 100 + 100 \cdot (2r_2 - 0.015) \cdot a_{2n|0.03} \]
\[ P_2 = 108 = 100 + 100 \cdot (r_2 - 0.015) \cdot a_{2n|0.03} \]
\[ \therefore \frac{32}{8} = \frac{2r_2 - 0.015}{r_2 - 0.015} \Rightarrow r_1 = 0.045, r_2 = 0.0225 \]
Ex 4.1.9

• A 7 % bond has a price of 79.30 and a 9 % bond has a price of 93.10, both per 100 of face amount. Both are redeemable in n years and have the same yield rate. Find n
Ex 4.1.9 soln

\[
\frac{P_1 - F}{P_2 - F} = \frac{F \cdot (r_1 - j) \cdot a_{n|j}}{F \cdot (r_2 - j) \cdot a_{n|j}}
\]

\[
\frac{79.30 - 100}{93.10 - 100} = \frac{0.035 - j}{0.045 - j} \implies j = 0.05
\]

\[
\Rightarrow 79.30 = \frac{100}{(1.05)^n} + 3.5 \cdot \left( \frac{1 - (1.05)^{-n}}{0.05} \right)
\]

\[
\Rightarrow (1.05)^{-n} = 0.31 \implies n = 24 \text{ periods} = 12 \text{ years}
\]
Ex 4.1.11

• Smith purchases a 20 year, 8 %, 1000 bond with semiannual coupons. The purchase price will give a nominal annual yield to maturity, convertible semiannually, of 10 %. After the 20th coupon, Smith sells the bond. At what price did he sell the bond if his actual nominal annual yield is 10 %?
Ex 4.1.11 soln

\[ P_0 = \frac{1000}{(1.05)^{40}} + 40 \cdot a_{40|0.05} = 828.41 \]

Consider sale price \( X \) as face value

\[ \therefore 828.41 = \frac{X}{(1.05)^{20}} + 40 \cdot a_{20|0.05} \]

\[ \Rightarrow X = 875.38 \]
You have decided to invest in two bonds. Bond X is a 2n-year bond with semi-annual coupons, while bond Y is an accumulation (zero-coupon) bond redeemable in n years. The desired yield rate is the same for both bonds. You also have the following information:

Bond X:
- Par value is 1000
- The ratio of the semi-annual bond rate to the desired semi-annual yield rate, r/i, is 1.03125.
- The present value of the redemption value is 381.50

Bond Y:
- Redemption value is the same as the redemption of Bond X
- Price to yield is 647.80

What is the price of Bond X?
Ex 4.1.19 Soln

\[ i = \text{yield per 6 month period} \]
\[ C = \text{common redemption value} \]

\[ 381.50 = \frac{C}{(1+i)^{2n}} \]

\[ Y = 647.80 = \frac{C}{(1+i)^{n}} \]

\[ \therefore \frac{1}{(1+i)^{n}} = \frac{381.50}{647.80} = 0.588916 \Rightarrow C = 1100 \]

\[ X = \frac{1100}{(1+i)^{2n}} + 1000 \cdot r \cdot a_{2n|i} = 1100 \cdot (0.588916)^2 + 1000 \cdot 1.03125 \cdot \left(1 - (0.588916)^2\right) \]

\[ = 1055 \]