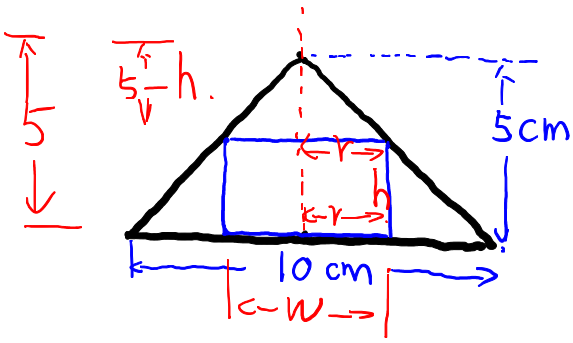
Name: _____ **ID:** _____

Complete the problems for 20 points. The scores will be recorded as Quiz9 and Quiz10 (10 pts each).

1. (8 points) Give an isosceles triangle with base 10cm and height 5cm. A rectangle is inscribed with its base on the base of the isosceles triangle and its upper corners on the two legs (the two two equal sides). What are the dimensions of such a rectangle with the greatest possible area? Find the greatest possible area.



Similar triangle:

$$\frac{5-h}{5} = \frac{w}{10}$$

$$\Rightarrow w = 10 \cdot \frac{5-h}{5} = 2(5-h)$$

$$A = w \cdot h = 2(5-h)\cdot h, \quad 0 \le h \le 5.$$

$$= 2(5h - h^2)$$

$$= 10h - 2h^2$$

$$A' = 10 - 4h = 0, \quad h = \frac{10}{4} = \frac{5}{2}$$

h	0	$\frac{5}{2}$	5
A	0	$10\cdot\frac{5}{2} - 2\left(\frac{5}{2}\right)^2$	0

Max area $= 10\cdot\frac{5}{2} - 2\cdot\left(\frac{5}{2}\right)^2 = 25 - \frac{25}{2} = \frac{25}{2}$

obtained at $h = \frac{5}{2}$

$w = 2\left(5 - \frac{5}{2}\right) = 5.$

2. (2 points) Find the most general anti-derivative of

$$f(x) = 4\cos x + 8 - \frac{\tan x \sec x}{3}.$$

anti-D: $4\sin x + 8x - \frac{\sec x}{3} + C$

3. (3 points) Find y (as a function of x) if $y' = \sin x$ and $y(0) = 2$.

anti-D of $\sin x$ is $-\cos x$:

$y = -\cos x + C$.

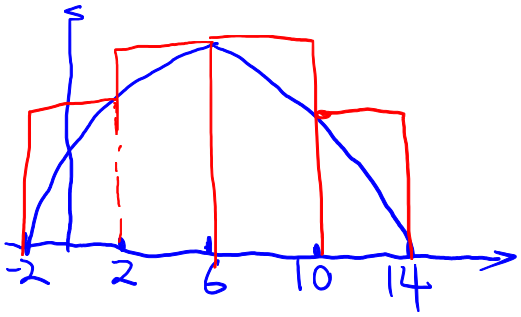
$y = -\cos x + 3$

$x=0,\ y=2$. $2 = -\cos 0 + C$

$2 = -1 + C \Rightarrow C = 3$

4. (4 points) Estimate the area under the graph of $f(x) = 28 + 12x - x^2$ from $x = -2$ to $x = 14$. Find the overestimate of the area using the "upper sum" of the areas of 4 rectangles of equal width.

graph of $y = f(x)$. y intercept: $y = 28$.



x intercepts: $28 + 12x - x^2 = 0$

$(14 - x)(2 + x) = 0$

$x = 14,\ x = -2$

Overestimate: $4 \cdot [f(2) + f(6) + f(6) + f(10)]$

$= 4(48 + 64 + 64 + 48) = 896$

$f(2) = 28 + 24 - 4 = 48$

$f(6) = 28 + 72 - 36 = 64$

$f(10) = 28 + 120 - 100 = 48$

5. (3 points) Find the sum

$$\sum_{i=0}^{2} (3^i - 1)$$

$= (3^0 - 1) + (3^1 - 1) + (3^2 - 1)$

$= 1 - 1 + 3 - 1 + 9 - 1$

$= 0 + 2 + 8$

$= 10$

_____ / 20