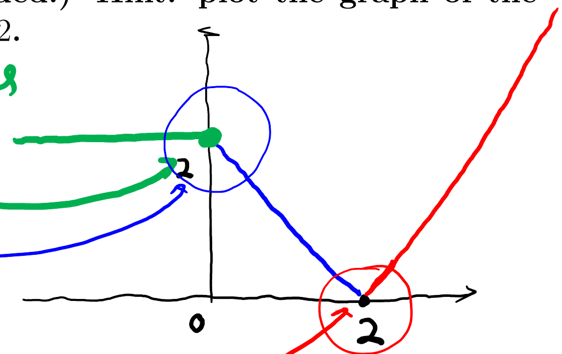
Name: ______________________________ **ID:** ______________

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish the test for 10 points.

The one on the back worth two extra points. Maximum of 10 points will be recorded for each quiz.

1. Let $f(x) = \begin{cases} 2 & x \le 0 \\ |x-2| & x > 0 \end{cases}$ $\begin{cases} -(x-2), & x-2<0 \\ x-2, & x-2 \geq 0 \end{cases}$ $f(x) = \begin{cases} 2 & x \le 0 \\ -(x-2) & 0<x<2 \\ x-2 & x \geq 2 \end{cases}$

Answer the following questions. (No explanation needed.) Hint: plot the graph of the piecewise function $y = f(x)$ **for** $x \le 0$, $0 < x < 2$ **and** $x \ge 2$.



(a) **(2 points) Is** $f(x)$ **continuous at** $x = 0$? continuous

Yes.

(b) **(2 points) Is** $f(x)$ **differentiable at** $x = 0$?

No Sharp Turn.

(c) **(2 points) Is** $f(x)$ **differentiable at** $x = 2$?

No Sharp Turn

2. (4 points) Use <u>THE LIMIT DEFINITION OF DERIVATIVE</u>

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

to find the derivative function $f'(x)$ **of** $f(x) = \frac{1}{2+x}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2+(x+h)} - \frac{1}{2+x}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2+x}{(2+x+h)(2+x)} - \frac{2+x+h}{(2+x)(2+x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2+x-(2+x+h)}{(2+x+h)\cdot(2+x)}}{h}$$

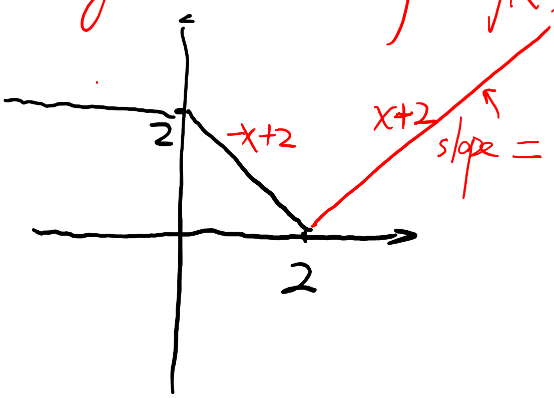
$$= \lim_{h\to 0} \frac{-\cancel{h}}{(2+x+h)(2+x)\cdot\cancel{h}} = \lim_{h\to 0} \frac{-1}{(2+x+h)(2+x)} \ \underset{h=0}{\text{Plug in}} \ \boxed{\frac{-1}{(2+x)(2+x)}}$$

(★ **Finish the problems on the front page first. No more than 10 points may be earned on the quiz. The extra problem is of average (or above) actual exam difficulty level. It is recommended to do it now or later to check whether you handle the materials well enough for the exam.)**
[2 extra points]

- **In problem 1, find**
$$\lim_{h\to 0+} \frac{f(2+h)-f(2)}{h}$$

compare to the limit definition $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

right derivative of $f(x)$ at $x=2$.



slope $= \lim_{h\to 0^+} \frac{f(2+h)-f(2)}{h} = \boxed{+1}$.

- **In problem 2, find the equation of the tangent line of the curve of** $f(x) = \frac{1}{2+x}$ **at the point** $(1, \frac{1}{3})$.

By ※2, $f'(x) = \frac{-1}{(2+x)^2}$

Plug in $x=1$. $f'(1) = \frac{-1}{(2+1)^2} = \frac{-1}{9}$.

Point-Slope Formula: $y - y_0 = k\cdot(x - x_0)$

$k = \text{slope} = f'(1) = \frac{-1}{9}$.

Point: $(x_0, y_0) = (1, \frac{1}{3})$.

Tangent line equation: $\boxed{y - \frac{1}{3} = \frac{-1}{9}(x-1)}$

______ / 10