Practice Final, Sec18

Multiple Choice Problems.

1. Suppose $f(x)$ is a continuous function with values given by the table below.

x	-2	-1	0	1
f(x)	0	3	0	-3

Which of the following statement is correct?

A $f(x) = 2$ has a root $c \in (-1, 0)$.

B $f(x) = 2$ has a root $c \in (0, 1)$.

C $f(x) = 4$ has a root $c \in (-1, 0)$.

D $f(x) = 4$ has a root $c \in (-2, 1)$.

E None of the above

2. Suppose you are estimating the root of $x^3 = 5x - 1$ using Newton's method. If you use $x_1 = 2$, find the exact value of x_2

A $x_2 = 2 - \frac{1}{7}$

B $x_2 = 2 + \frac{1}{7}$

C $x_2 = 8 - \frac{8}{9}$

D $x_2 = 8 + \frac{8}{9}$

E $x_2 = 5 + \frac{1}{7}$

3. Evaluate the limit:

$$\lim_{x \to 3} \frac{x+2}{x(x-3)}$$

A $+\infty$

B $-\infty$

C $\frac{5}{3}$

D $-\frac{5}{3}$

E The limit does not exist.

4. Find the horizontal aysmptote(s) of the following function:

$$f(x) = \frac{x-2}{3x+5}$$

A $x = \frac{1}{3}$

B $y = \frac{1}{3}$

C $x = -\frac{5}{3}$

D $y = 2$

E $y = -\frac{2}{5}$

5. Compute the limit:

$$\lim_{h\to 0} \frac{\frac{1}{h+2} - \frac{1}{2}}{h}$$

A $+\infty$

B $\frac{1}{2}$

C $\frac{1}{4}$

D $-\frac{1}{4}$

E 0

6. Find the limit:

$$\lim_{x\to 0} \frac{\sin(2x)}{3x}$$

A $\frac{2}{3}$

B $\frac{3}{2}$

C 0

D ∞

E Does not exist.

7. Suppose $\int_0^2 f(x)\ dx = -4$, $\int_0^5 f(x)\ dx = 6$. Find $\int_2^5 f(x)dx$ and the average of $f(x)$ over $[2, 5]$

A $\int_2^5 f(x)dx = 2$, average of f is $\frac{2}{3}$

B $\int_2^5 f(x)dx = 10$, average of f is $\frac{10}{3}$

C $\int_2^5 f(x)dx = -10$, average of f is $-\frac{10}{3}$

D $\int_2^5 f(x)dx = -2$, average of f is $-\frac{2}{3}$

E $\int_2^5 f(x)dx = 10$, average of f is $\frac{10}{5}$

8. Evaluate

$$\int_{-\pi}^{\pi} \sin x \cdot \sqrt{\cos x + 2}\ dx$$

A $\frac{4}{3}$

B 0

C $-\frac{4}{3}$

D $-\frac{2}{3}$

E 2

9. Evaluate the sum

$$\sum_{i=1}^{20} \frac{4-i}{2}$$

A $40 - \frac{20\times 21}{2}$
B $40 - \frac{20\times 21}{4}$
C $20 - \frac{20\times 21}{2}$
D $20 - \frac{20\times 21}{4}$
E $\frac{20\times 21}{2}$

10. Evaluate the integral

$$\int \sqrt[3]{2x-8}\, \mathrm{d}x$$

A $\frac{3}{4}x^{\frac{4}{3}} + C$
B $\frac{3}{8}(2x-8)^{\frac{4}{3}} + C$
C $\frac{3}{4}(2x-8)^{\frac{4}{3}} + C$
D $\frac{3}{8}x^{\frac{4}{3}} + C$
E $\frac{1}{3}(2x-8)^{\frac{3}{2}} + C$

11. Find the average value of $f(x) = 2x + 3$ on $[-1, 2]$;

A 4
B 12
C $\frac{8}{3}$
D -4
E 8

12. Sovle the initial value problem if

$$y' = \sin(\frac{x}{3}), \quad y(0) = 4$$

A $-3\cos(\frac{x}{3}) + 1$
B $-\cos(\frac{x}{3}) + 7$
C $-3\cos(\frac{x}{3}) + 7$
D $-\frac{1}{3}\cos(\frac{x}{3}) + 1$
E $-3\sin(\frac{x}{3}) + 4$

Standard Response Problems.

1. Calculate the first and second order derivatives of $f(x) = x \sin x$. And find the equation of the tangent line to the curve $y = f(x)$ at $x = 0$

2. Find the derivatives of

$$f(x) = \frac{\cos(x^2)}{\sqrt{x}}$$

3. Suppose that y and x satisfy the implicit equation

$$xy^3 + xy = 20$$

(a) Find $\frac{dy}{dx}$

(b) Use your answer in part (a) to find the equation of the tangent line to the curve $xy^3 + xy = 20$ at the point $(10, 1)$.

4. If the radius of a circular ink blot is growing at a rate of 3 cm/min. How fast (in cm^2/min) is the area of the blot growing when the radius is 10 cm?

5. Car A is traveling west at 50 mi/h and car B is traveling north at 60mi/h. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?

6. Find the absolute maximum and minimum of $f(x) = -x^3 + 3x$ on $[-1, 2]$.

7. A particle moves with velocity $v(t) = -t^2 + 6t - 8$, $0 \leq t \leq 6$. Sketch the graph of $v(t)$ on $[2, 4]$. USE **FOUR RECTANGLES OF EQUAL WIDTH** to find the overestimate of the displacement of the particle traveled from $t = 2$ to $t = 4$.

8. (S16) Suppose $f(x) = x^4 - 6x^2 - 3$.

   (a) Identify the intervals over which $f(x)$ is increasing and decreasing, and all values of x where $f(x)$ attains its local maximum or minimum.

   (b) Identify the intervals over which $f(x)$ is concave up and down, and all values of x where $f(x)$ has an inflection point.

9. Calculate the integral

$$\int \frac{x^2}{\sqrt{3+x^3}}\, dx$$

10. Calculate the integral $\int_0^{\pi/4} \tan x \cdot \sec x + 2x \; dx$

11. Find the area of the region enclosed by the graphs of the equations $y = x + 4$ and $y = x^2 - x + 1$.